**Anticipation Guide**

**Factoring**

**Step 1**  
Before you begin Chapter 8

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

<table>
<thead>
<tr>
<th>STEP 1</th>
<th>Statement</th>
<th>STEP 2</th>
<th>A or D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, D, or NS</td>
<td>A. A monomial is in factored form when it is expressed as the product of prime numbers and variables, and no variable has an exponent greater than 1.</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B. The greatest common factor (GCF) of two or more monomials is the product of their unique factors when each monomial is written in factored form.</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C. Any two numbers that have a greatest common factor of 1 are said to be relatively prime.</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>D. If the product of any two factors is 0, then at least one of the factors must equal 0.</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>E. A quadratic trinomial has a degree of 2.</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F. To solve an equation such as ( x^2 + 2x ), take the square root of both sides.</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td></td>
<td>G. The polynomial ( 3x^2 - r - 2 ) cannot be factored because the coefficient of ( x^2 ) is not 1.</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td></td>
<td>H. The polynomial ( 3x^2 + 16 ) is not factored.</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>I. The numbers 16, 64, and 121 are perfect squares.</td>
<td>A</td>
<td></td>
</tr>
</tbody>
</table>

**Step 2**  
After you complete Chapter 8

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

**8-1 Study Guide and Intervention**

**Monomials and Factoring**

**Factor Monomials**  
A monomial is in factored form when it is expressed as the product of prime numbers and variables, and no variable has an exponent greater than 1.

**Example**

Factor each monomial completely.

a. \( 42a^2 \)

\[
42a^2 = 2 \cdot 21 \cdot a \cdot a \\
= 2 \cdot 3 \cdot 7 \cdot a \cdot a \\
= 21 \cdot 3 \cdot a \cdot a \\
= 21 \cdot 3 \cdot a \cdot a \\
\]

Thus, \( 42a^2 \) in factored form is \( 2 \cdot 3 \cdot 7 \cdot a \cdot a \).

b. \(-40x^2y^3\)

\[
-40x^2y^3 = -1 \cdot 40 x^2 y^3 \\
= -1 \cdot 2 \cdot 20 \cdot x \cdot x \cdot y \cdot y \cdot y \\
= -1 \cdot 2 \cdot 2 \cdot 10 \cdot x \cdot x \cdot y \cdot y \cdot y \\
= -1 \cdot 2 \cdot 2 \cdot 5 \cdot x \cdot x \cdot y \cdot y \\
= -1 \cdot 2 \cdot 2 \cdot 5 \cdot x \cdot x \cdot y \cdot y \\
\]

Thus, \(-40x^2y^3\) in factored form is \(-1 \cdot 2 \cdot 2 \cdot 5 \cdot x \cdot x \cdot y \cdot y \).

**Exercises**

Factor each monomial completely.

1. \(12t^2\)

\[
12t^2 \quad \text{or} \quad 2 \cdot 2 \cdot 3 \cdot t \cdot t \\
\]

2. \(18m^2n\)

\[
18m^2n \quad \text{or} \quad 2 \cdot 2 \cdot 3 \cdot m \cdot m \cdot n \\
\]

3. \(45k^2b^2\)

\[
45k^2b^2 \quad \text{or} \quad 3 \cdot 5 \cdot k \cdot k \cdot b \cdot b \\
\]

4. \(16h^8\)

\[
16h^8 \quad \text{or} \quad 2 \cdot 2 \cdot 2 \cdot 2 \cdot h \cdot h \cdot h \cdot h \\
\]

5. \(-9h^3k^4\)

\[
-9h^3k^4 \quad \text{or} \quad -1 \cdot 3 \cdot 3 \cdot h \cdot h \cdot k \cdot k \\
\]

6. \(-8t^7\)

\[
-8t^7 \quad \text{or} \quad -1 \cdot 2 \cdot 2 \cdot 2 \cdot t \cdot t \cdot t \cdot t \\
\]

7. \(64y^8\)

\[
64y^8 \quad \text{or} \quad 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot y \cdot y \cdot y \cdot y \\
\]

8. \(9ab^6f^2\)

\[
9ab^6f^2 \quad \text{or} \quad 3 \cdot 3 \cdot a \cdot b \cdot b \cdot f \cdot f \\
\]

9. \(-17t^7\)

\[
-17t^7 \quad \text{or} \quad -1 \cdot 17 \cdot t \cdot t \cdot t \cdot t \cdot t \cdot t \\
\]

10. \(125k^4\)

\[
125k^4 \quad \text{or} \quad 5 \cdot 5 \cdot 5 \cdot j \cdot k \cdot k \\
\]

11. \(47w^4x^2y^2z^2\)

\[
47w^4x^2y^2z^2 \quad \text{or} \quad 47 \cdot w \cdot x \cdot x \cdot y \cdot y \cdot z \cdot z \\
\]

12. \(-32a^4b^4\)

\[
-32a^4b^4 \quad \text{or} \quad -1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot a \cdot a \cdot b \cdot b \\
\]

13. \(2 \cdot 2 \cdot 2 \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \\)
8-1 Study Guide and Intervention (continued)

Monomials and Factoring

Greatest Common Factor The product of the common prime factors is called the greatest common factor (GCF) of the numbers. The greatest common factor is the greatest number that is a factor of both original numbers.

If two or more integers or monomials have no common prime factors, their GCF is 1 and the integers or monomials are said to be relatively prime.

Example Find the GCF of 16xy²z and 72xyz².

16xy²z = 2² · 2 · 2 · 2 · x · y · z
72xyz² = 2³ · 3 · 3 · x · y · z · z

The GCF of 16xy²z and 72xyz² is 2 · 2 · x · y · z or 8xyz.

Exercises
Find the GCF of each set of monomials.

1. 49x, 343x² 2. 4a³, 28ab
   49x 4a³
3. 9b², 12x, 8y 4. 12a, 18x³
   4 6a
5. 28y³, 35cy, 49cy²z 6. 2mn³, 12m³n, 18mn³
   7y 2mp
7. 12c³, 32c³y, 60xy³ 8. 18a³b³, 36a³b³
   4x 18a³b³
9. 15m³, 30m²n⁴, 90m⁵ 10. 2x³y, 9x³y³, 18x³y²
   15m 3xy
11. a³b, 8a³b² 11. a³b³
   a³b a³b³
12. 2x²y³, 8xy³, 12x²y² 12. 3a³b³
   2xy 3a³b³
13. 2ab⁴, 8xy³, 12x²y² 13. 12x³y², 5x³y², 10x³y²
   2xy xy
14. 9a³b³, 15a³b³ 14. 13x³y², 5x³y², x³y
   3a³b³ xy
15. 27a³b³, 84a³b³, 26a³b³ 15. 20r²t, 26r²t, 2r²t
   7a³b³ 7a³b³

Find the GCF of each set of monomials.

1. 10a³, 2 · 5 · a · a · a · a 2. 27x³y³, −1 · 3 · 3 · x · x · x · y · y
   10a³ 27x³y³
3. 28xy², 2 · 2 · 7 · p · r · r 4. 44m³np³, 2 · 2 · 11 · m · m · n · p · p · p
   4. 28xy² 44m³np³
5. 9x³y³, 3 · 3 · x · x · x · y · y 6. −17a²b³, −1 · 17 · a · b · b · f
   9x³y³ 17a²b³
7. 42x², 2 · 3 · 7 · g · g 8. 38a²u², 2 · 2 · 3 · t · u · u
   6 · 7 · x² 38a²u²
9. −4a · −1 · 2 · 2 · a 10. −10x³y³, −1 · 2 · 5 · x · x · y · z · z
   −4a −10x³y³

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8-1 Practice
Monomials and Factoring

Factor each monomial completely.

1. $30d^5$
2. $-72np$
3. $81b^6c^3$
4. $14ab^2c^2$
5. $168w^2y^2$
6. $-121x^2y^2z^2$
7. $-14q^2p^2$
8. $-7m^2n^2p^2q^2r^2$

Find the GCF of each set of monomials.
9. $24g^3, 56g^5, 8fg$
10. $72r^5, 36r^3, 36rt^3$
11. $15axb, 35axb, 5ab$
12. $25a^2b^2, 45xy^2, 1$
13. $40xy^3, 56x^2y, 124e^3y^2, 4xy^2$
14. $88a^2d, 40a^4d, 32a^6d, 8a^2d$

15. GEOMETRY The area of a rectangle is 84 square inches. Its length and width are both whole numbers.

a. What is the minimum perimeter of the rectangle? 38 in.
b. What is the maximum perimeter of the rectangle? 170 in.

16. RENOVATION Ms. Baxter wants to tile a wall to serve as a splashguard above a basin in the basement. She plans to use equal-sized tiles to cover an area that measures 48 inches by 36 inches.

a. What is the maximum-size square tile Ms. Baxter can use and not have to cut any of the tiles? 12-in. square
b. How many tiles of this size will she need? 12

8-1 Word Problem Practice
Monomials and Factoring

1. MATH GAMES Mrs. Jenson’s class is playing “Guess the Monomial.” One student displays factors of the secret monomial, and the team tries to guess the monomial. When it is James’ turn, he sees that the secret monomial is $210x^5y^3$. Which of the following cards should he display so his team guesses the correct monomial?

Order may vary:
$2 \cdot 3 \cdot 5 \cdot 7 \cdot x \cdot x \cdot y \cdot y$

2. PARTY FAVORS Balloons come in packages of 18 and party hats come in packages of 8. Jeff wants to have the same number of balloons and hats. What is the fewest packages of balloons and hats that he needs to buy so he has no hats or balloons left over? 4 packages of balloons and 9 packages of hats

3. PACKAGING Color Wheel printer ink company wants to design a new carton in which to pack printer ink cartridges for shipment to stores. Cartridge boxes are 7 inches long and 3 inches wide. What are the dimensions of the smallest square-bottom carton that will hold the cartridge boxes without extra space? 21 in. by 21 in.

4. MATHEMATICIANS A Greek mathematician and astronomer named Eratosthenes created a way to separate prime numbers from composite numbers. His method is known as the Sieve of Eratosthenes. It proceeds as follows.

Write numbers 1 to 50. Since 1 is neither prime nor composite, ignore 1. Circle the number 2, and then cross off every multiple of 2. Circle the next number that is not crossed off, 3, and cross off all multiples of 3. Circle the next number that is not crossed off, 5, and cross off all multiples of 5, etc…

5. REPAIRS Heidi wants to replace the floor in her 16-foot by 18-foot rectangular dance studio. She wants to use square wood tiles, and she does not want to have to cut any of the tiles nor leave any gaps.

a. Suppose the flooring company can use any size tile. What is the largest square tile that Heidi can use for the new floor? 2 feet by 2 feet
b. If Heidi first knocks out a wall and increases the studio to 24 feet by 18 feet, what is the largest square tile she can use for the new floor? 6 foot by 6 foot
Chapter 8

8-1 Enrichment

Finding the GCF by Euclid’s Algorithm

Finding the greatest common factor of two large numbers can take a long time using prime factorizations. This method can be avoided by using Euclid’s Algorithm as shown in the following example.

Example
Find the GCF of 209 and 532.
Divide the greater number, 532, by the lesser, 209.

\[
\begin{array}{c|c}
2 & 209 \\
\hline
41 & 532 \\
\hline

\end{array}
\]

Divide the remainder into the divisor above.

\[
\begin{array}{c|c}
114 & 532 \\
\hline
1 & 209 \\
\hline

\end{array}
\]

Repeat this process until the remainder is zero. The last nonzero remainder is the GCF.

\[
\begin{array}{c|c}
5 & 114 \\
\hline
95 & 114 \\
\hline
5 & 95 \\
\hline
95 & 5 \\
\hline
0 & 95 \\
\hline
\end{array}
\]

The divisor, 19, is the GCF of 209 and 532.

Suppose the GCF of two numbers is found to be 1. Then the numbers are said to be relatively prime.

Find the GCF of each group of numbers by using Euclid’s Algorithm.

1. 187; 578 17
2. 1802; 106 106
3. 161; 943 23
4. 215; 1849 43
5. 1525; 3498 53
6. 3484; 5963 67
7. 33,563; 4257 473
8. 453; 484 1 (relatively prime)
9. 95; 209; 589 19
10. 518; 407; 851 37
11. 17x^2z; 161x^2z 17xz
12. 753y^3; 890x^2y^3 471ax^2
13. 979r^4; 495x^2r^4 11r^2
14. 360a^2y^3; 328a^2y^3; 568s^2y^3 8xy

Answers

Chapter 8

8-2 Study Guide and Intervention

Using the Distributive Property

Use the Distributive Property to Factor

The Distributive Property has been used to multiply a polynomial by a monomial. It can also be used to express a polynomial in factored form. Compare the two columns in the table below.

<table>
<thead>
<tr>
<th>Multiplying</th>
<th>Factoring</th>
</tr>
</thead>
<tbody>
<tr>
<td>3ax + 6b</td>
<td>3a(x + 2b)</td>
</tr>
<tr>
<td>7xy - 5yz</td>
<td>7(x - y)z</td>
</tr>
<tr>
<td>6(ax + 1)</td>
<td>6(a + 1)x</td>
</tr>
<tr>
<td>12a + by</td>
<td>12(a + b)y</td>
</tr>
</tbody>
</table>

Example 1
Use the Distributive Property to factor 12mp + 80m^2.
Find the GCF of 12mp and 80m^2.
12mp = 2 · 2 · 3 · m · p
80m^2 = 2 · 2 · 2 · 2 · 2 · 5 · m · m
GCF = 2 · 2 · m or 4m
Write each term as the product of the GCF and its remaining factors.
12mp + 80m^2 = 4m(3p) + 4m(20m)
= 4m(3p) + 4m(20m)
= 4m(3p + 20m)
Thus 12mp + 80m^2 = 4m(3p + 20m).

Example 2
Factor 6ax + 3ay + 2bx + by by grouping.
6ax + 3ay + 2bx + by = (6ax + 3ay) + (2bx + by)
= 3a(2x + y) + b(2x + y)
= (3a + b)(2x + y)
Check using the FOIL method.
(3a + b)(2x + y) = 3ax + 3ay + 2bx + by
Thus 6ax + 3ay + 2bx + by = (3a + b)(2x + y).

Exercises

Factor each polynomial.

1. 24x + 48y
2. 30mp^2 + mp + 6p
3. q^2 - 18q^2 + 22q
24(x + 2y)
(30mp^2 + mp + 6p)
(q^2 - 18q^2 + 22q)

4. 5x^2 - 3x
5. 4m + 6p + 8mp
6. 45r^2 - 15t^2
3x(3x - 1)
2(2m + 3p + 4mp)
15r^2(3r - 1)

7. 14t^2 - 42t + 49t
8. 55p^3 - 11p + 44p^2
9. 14y^2 - 28y^2 + 1
7(2t - 6)^2 + 7t
11p^2(5 - p^2 + 4p^2)
y(14y^2 - 28y^2 + 1)

10. 4x + 12x^2 + 16x^3
11. 4a + b + 28x + b^3 + 7b
12. 6y + 12x - 8z
4x(1 + 3x + 4x^2)
ab(4a + 28b + 7b)
2(3y + 6x + 2z)

13. x^2 + 2x + x + 2
14. 6x^2 - 4y + 3y - 2
15. 14m^2 + 4np + 3np + 3p^2
(x + 1)(x + 2)
(2y + 1)(3y - 2)
(4m + 3p)(m + p)

16. 12a + 3ax + 4ax + yz
17. 13a^2 + 3a - 8a^2 - 2
18. ax + ay + x + y
(3x + y)(4a + z)
(4a + 1)(3a - 2)
(x + y)(a + 1)
8-2 Study Guide and Intervention (continued)

Using the Distributive Property

Solve Equations by Factoring

The following property, along with factoring, can be used to solve certain equations.

Zero Product Property

For any real numbers a and b, if ab = 0, then either a = 0, b = 0, or both a and b equal 0.

Example

Solve 9x^2 + x = 0. Then check the solutions.

Write the equation so that it is of the form ab = 0.

9x^2 + x = 0

Factor the GCF of 9x^2 + x, which is x.

x = 0 or 9x + 1 = 0

Zero Product Property

x = 0 or x = -1/9

Solve each equation.

The solution set is \{0, -1/9\}.

Check

Substitute 0 and -1/9 for x in the original equation.

9x^2 + x = 0

9(0)^2 + 0 = 0

0 = 0 \checkmark

9\left(-\frac{1}{9}\right)^2 + 9\left(-\frac{1}{9}\right) = 0

0 = 0 \checkmark

Exercises

Solve each equation. Check your solutions.

1. x(x + 3) = 0
   \{0, -3\}

2. 3m(m - 4) = 0
   \{0, 4\}

3. (r - 3)(r + 2) = 0
   \{-2, 3\}

4. 3x^2x - 1 = 0
   \{0, \frac{1}{2}\}

5. 4(m - 8)(m - 3) = 0
   \{-2, 3\}

6. 5x^2 = 25m
   \{0, 5\}

7. 4x^2 + 2(3x - 7) = 0
   \{9, \frac{1}{3}\}

8. 5p - 15p = 0
   \{0, 2\}

9. 4x^2 = 28
   \{0, 7\}

10. 12x^2 = -6x
    \{-\frac{1}{2}, 0\}

11. 6x^2 + 3x + 2 = 0
    \{-\frac{1}{2}, 0\}

12. 8y^2 - 12y
    \{-\frac{1}{2}, 0\}

13. x^2 = -2x
    \{-\frac{1}{2}, 0\}

14. 6y - 4(y + 3) = 0
    \{-2, 0\}

15. 2x^2 + 2x + 3
    \{0, 1\}

16. 12x^2 = 3x^2
    \{0, 4\}

17. 12a^2 = 3a
    \{0, 4\}

18. 12x^2 + 4|3a - 1| = 0
    \{-1, \frac{1}{3}\}

19. x(x - 3) = 0
    \{0, 3\}

20. 6b + 12 = 0
    \{-2, 0\}

21. (m - 3)(m + 5) = 0
    \{-5, 3\}

22. x - 9 = 2ax + 1
    \{-\frac{1}{2}, 9\}

23. x^2 = 3x
    \{0, 5\}

24. 3y^2 + 3y = 0
    \{-3, 0\}

25. 3x^2 = 6a
    \{0, 2\}

26. 2x^2 = 3x
    \{0, \frac{3}{2}\}
8-2 Practice

Using the Distributive Property

Factor each polynomial.

1. $64 - 40b + 8(5 - 6b)$
2. $4x^2 + 16$
3. $6r^2 - 3r^2$
4. $15a^2 + 30a^2d^2$
5. $5x^2(6x + 7y)$
6. $8p(3x^2 + 6x^2)$
7. $3b^2 + 35b^2$
8. $7r^2 - 6r^2$

Solve each equation. Check your solutions.

19. $x(x - 32) = 0$
20. $4b(b + 4) = 0$
21. $(y - 3)(y + 2) = 0$

1. PHYSICS According to legend, Galileo dropped objects of different weights from the so-called "leaning tower" of Pisa while developing his formula for free falling objects. The relationship that he discovered was that the distance $d$ an object falls after $t$ seconds is given by $d = 16t^2$ (ignoring air resistance). This relationship can be found in the equation $h = 4t^2$, where $h$ is the height of an object thrown upward from ground level at a rate of 32 feet per second. Solve the equation for $h = 0$. $t = 0.25$ and 0

3. CONSTRUCTION Unique Building Company is constructing a triangular roof truss for a building. The workers assemble the truss with the dimensions shown on the diagram below. Using the Pythagorean Theorem, find the length of the sides of the truss. 3 yd, 4 yd, 5 yd

4. VERTICAL JUMP Your vertical jump height is measured by subtracting your standing reach height from the height of the highest point you can reach by jumping without taking a running start. Typically, NBA players have vertical jump heights of up to 34 inches. If an NBA player jumps this high, his height $h$ in inches above his standing reach height after $t$ seconds can be modeled by the equation $h = 10t^2 - 10t$. Solve the equation for $h = 0$ and interpret the solution. Round your answer to the nearest hundredth.

5. PETS Conner tosses a dog treat upward with an initial velocity of 13.7 meters per second. The height of the treat above the dog's mouth $h$ in meters after $t$ seconds is given by the equation $h = 13.7t - 4.9t^2$

a. Assuming the dog doesn't jump, after how many seconds does the dog catch the treat? 2.795
b. The dog treat reaches its maximum height halfway between when it was thrown and when it was caught. What is its maximum height? 9.6 m
c. How fast would Conner have to throw the dog treat in order to make it fly through the air for 6 seconds? 29.4 m/s
**Linear Combinations**

The greatest common factor, GCF, of two numbers can be written as a linear combination of the two numbers. A linear combination is an expression of the form $Ax + By$.

**Example**

Write the greatest common factor of 52 and 36 as a linear combination.

First, use the Euclidean Algorithm to find the greatest common factor of the two numbers.

1. Divide the greater number by the lesser number.
2. Use the remainder as the new divisor.
3. Stop dividing when the remainder is 0.

Here is the process for finding the GCF of 52 and 36:

\[
\begin{align*}
36 & ) 52 \\
2 & \text{original divisor; divide again.} \\
16 & \text{second divisor; divide again.} \\
0 & \text{Last divisor used is the GCF.} \\
\end{align*}
\]

The GCF is 4.

To write 4 as a linear combination of 36 and 52, it needs to be written as:

\[4 = 36(1) + 52(-2)\]

**Exercises**

Write the greatest common factor for each pair of numbers as a linear combination.

1. 16, 64
   \[16 = 16(1) + 64(0)\]
2. 21, 28
   \[7 = (1-1) + 28(1)\]
3. 3, 18
   \[3 = 3(1) + 18(0)\]
4. 15, 36
   \[3 = 15(5) - 36(2)\]
5. 6, 8
   \[2 = 6(-1) + 8(1)\]
6. 18, 42
   \[6 = 18(-2) + 42(1)\]

**8-3 Study Guide and Intervention**

### Quadratic Equations: $x^2 + bx + c = 0$

Factor $x^2 + bx + c$ To factor a trinomial of the form $x^2 + bx + c$, find two integers, $m$ and $p$, whose sum is equal to $b$ and whose product is equal to $c$.

**Example 1**

Factor each polynomial.

\[\begin{align*}
a. \quad x^2 + 7x + 10 & \quad \text{In this trinomial, } b = 7 \text{ and } c = 10. \\
\text{Factors of 10} & \quad \text{Sum of Factors} \\
1, 10 & \quad 11 \\
2, 5 & \quad 7 \\
\text{Since } 2 + 5 = 7 \text{ and } 2 \cdot 5 = 10, \text{ let } m = 2 \text{ and } p = 5. \\
x^2 + 7x + 10 = (x + 5)(x + 2) \\
b. \quad x^2 - 8x + 7 & \quad \text{In this trinomial, } b = -8 \text{ and } c = 7. \\
\text{Notice that } m + p \text{ is negative and } mp \text{ is positive, so } m \text{ and } p \text{ are both negative.} \\
\text{Since } -7 + (-1) = -8 \text{ and } (-7)(-1) = 7, \\
m = -7 \text{ and } p = -1. \\
x^2 - 8x + 7 = (x - 7)(x - 1) \\
\end{align*}\]

**Example 2**

Factor $x^2 + 6x - 16$.

In this trinomial, $b = 6$ and $c = -16$. This means $m + p$ is positive and $mp$ is negative. Make a list of the factors of $-16$, where one factor of each pair is positive.

\[\begin{align*}
\text{Factors of } -16 & \quad \text{Sum of Factors} \\
1, -16 & \quad -15 \\
-1, 16 & \quad 15 \\
2, -8 & \quad -6 \\
-2, 8 & \quad 6 \\
\text{Therefore, } m = -2 \text{ and } p = 8. \\
x^2 + 6x - 16 = (x - 2)(x + 8) \\
\end{align*}\]
Study Guide and Intervention (continued)

Quadratic Equations: \( x^2 + bx + c = 0 \)

Solve Equations by Factoring. Factoring and the Zero Product Property can be used to solve many equations of the form \( x^2 + bx + c = 0 \).

**Example 1** Solve \( x^2 + 6x = 7 \). Check your solutions.

\[
\begin{align*}
&x^2 + 6x = 7 \quad \text{Original equation} \\
&x^2 + 6x - 7 = 0 \quad \text{Rewrite equation so that one side equals 0.} \\
&(x - 1)(x + 7) = 0 \quad \text{Factor.} \\
&x - 1 = 0 \quad \text{or} \quad x + 7 = 0 \quad \text{Zero Product Property} \\
&x = 1 \quad \text{or} \quad x = -7 \quad \text{Solve each equation.}
\end{align*}
\]

The solution set is \( \{1, -7\} \). Since \( 1^2 + 6 = 7 \) and \( (-7)^2 + 6(-7) = 7 \), the solutions check.

**Example 2** ROCKET LAUNCH. A rocket is fired with an initial velocity of 2288 feet per second. How many seconds will it take for the rocket to hit the ground?

The formula \( h = vt - 16t^2 \) gives the height \( h \) of the rocket after \( t \) seconds when the initial velocity \( v \) is given in feet per second.

\[
\begin{align*}
&h = vt - 16t^2 \quad \text{Formula} \\
&0 = 2288t - 16t^2 \quad \text{Substitute.} \\
&16t = 0 \quad \text{or} \quad 143 - t = 0 \quad \text{Zero Product Property} \\
&t = 0 \quad \text{or} \quad t = 143 \quad \text{Solve each equation.} \\
\end{align*}
\]

The value \( t = 0 \) represents the time at launch. The rocket returns to the ground in 143 seconds, or a little less than 2.5 minutes after launch.

**Exercises**

Solve each equation. Check the solutions.

\[
\begin{align*}
1. x^2 - 4x + 3 &= 0 \quad \{1, 3\} \\
2. x^2 - 5x + 4 &= 0 \quad \{1, 4\} \\
3. m^2 + 10m + 9 &= 0 \quad \{-1, -9\} \\
4. x^2 = x + 2 \quad \{-1, 2\} \\
5. x^2 - 4x + 5 &= \{-1, 5\} \\
6. x^2 - 12x + 36 &= 0 \quad \{6\} \\
7. p^2 - 8p - 10 &= -8 - 1 \quad \{-8, 1\} \\
8. p^2 = 9p - 14 \quad \{2, 7\} \\
9. 9 - 9 - 8x + x^2 &= 0 \quad \{-1, 9\} \\
10. y^2 + 6y + 5 &= (2, 3) \\
11. a^2 = 11a - 18 \quad \{2, 9\} \\
12. y^2 - 8y + 15 &= 0 \quad \{3, 5\} \\
13. x^2 = 24 - 16x \quad \{-12, 2\} \\
14. a^2 - 18a = 72 \quad \{6, 12\} \\
15. b^2 = 10b - 16 \quad \{2, 8\}
\end{align*}
\]

Use the formula \( h = vt - 16t^2 \) to solve each problem.

16. FOOTBALL A punter can kick a football with an initial velocity of 48 feet per second. How many seconds will it take for the ball to return to the ground? \( 3 \) seconds

17. BASEBALL A ball is thrown up with an initial velocity of 32 feet per second. How many seconds will it take for the ball to return to the ground? \( 2 \) seconds

18. ROCKET LAUNCH If a rocket is launched with an initial velocity of 1600 feet per second, when will the rocket be 14400 feet high? at 10 seconds and at 90 seconds

**Skills Practice**

Factor each polynomial.

\[
\begin{align*}
1. x^2 + 8t + 12 &= (t + 2)(t + 6) \\
2. y^2 + 7n + 12 &= (n + 3)(n + 4) \\
3. p^2 + 9p + 20 &= (p + 5)(p + 4) \\
4. h^2 + 9h + 18 &= (h + 6)(h + 3) \\
5. r^2 + 3n - 18 &= (n + 6)(n - 3) \\
6. x^2 + 2t - 8 &= (x + 4)(x - 2) \\
7. y^2 - 5p - 6 &= (y + 1)(y - 6) \\
8. a^2 + 3p - 10 &= (g + 5)(g - 2) \\
9. x^2 + 4r - 12 &= (r - 2)(r + 6) \\
10. x^2 - x - 12 &= (x - 4)(x + 3) \\
11. w^2 - w - 6 &= (w - 3)(w + 2) \\
12. y^2 - 6y + 8 &= (y - 2)(y + 4) \\
13. x^2 - 8x + 15 &= (x - 3)(x - 5) \\
14. h^2 - 9h + 8 &= (b - 1)(b - 8) \\
15. e^2 - 16 + 56 &= 16 - 4 - 3m + m^2 \\
16. x^2 - 7b + 12 &= (m - 4)(m + 1)
\end{align*}
\]

Solve each equation. Check the solutions.

\[
\begin{align*}
17. x^2 - 6r + 8 &= 0 \quad \{2, 4\} \\
18. b^2 - 7b + 12 &= 0 \quad \{3, 4\} \\
19. m^2 + 5m + 6 &= 0 \quad \{-3, -2\} \\
20. a^2 + 7a + 10 &= 0 \quad \{-5, -2\} \\
21. y^2 - 2y - 24 &= 0 \quad \{-4, 6\} \\
22. p^2 - 3p = 18 \quad \{-3, 6\} \\
23. b^2 + 2b = 35 \quad \{-7, 5\} \\
24. a^2 + 4a = 45 \quad \{-9, -5\} \\
25. n^2 - 36 = 5n \quad \{-4, 9\} \\
26. m^2 + 30 = 11w \quad \{5, 6\}
\end{align*}
\]
8-3 Practice

Quadratic Equations: $x^2 + bx + c = 0$

Factor each polynomial.

1. $x^2 + 10x + 24 = 0$
   \[ (x + 4)(x + 6) \]

2. $h^2 + 12h + 27 = 0$
   \[ (h + 3)(h + 9) \]

3. $x^2 + 16x + 33 = 0$
   \[ (x + 11)(x + 3) \]

4. $x^2 + 3x + 2 = 0$
   \[ (x + 1)(x + 2) \]

5. $x^2 + 9x + 18 = 0$
   \[ (x + 3)(x + 6) \]

6. $x^2 + 5x - 6 = 0$
   \[ (x + 6)(x - 1) \]

7. $x^2 + 4h - 12 = 0$
   \[ (x - 3)(x + 4) \]

8. $n^2 - 3n - 28 = 0$
   \[ (n - 7)(n + 4) \]

9. $x^2 + 4t - 45 = 0$
   \[ (x - 9)(x + 5) \]

10. $x^2 - 11x + 30 = 0$
    \[ (x - 6)(x - 5) \]

11. $d^2 - 16d + 63 = 0$
    \[ (d - 9)(d - 7) \]

12. $x^2 - 11x + 24 = 0$
    \[ (x - 3)(x - 8) \]

13. $q^2 - q - 56 = 0$
    \[ (q - 8)(q + 7) \]

14. $x^2 - 6x - 55 = 0$
    \[ (x - 11)(x + 5) \]

15. $32 + 18r + r^2 = 0$
    \[ (r + 16)(r + 2) \]

16. $g^2 - 16g + 25 = 0$
    \[ (g - 5)(g - 5) \]

17. $j^2 - 9k - 10k^2 = 0$
    \[ (j - 10k)(j + k) \]

18. $m^2 - 5m - 60 = 0$
    \[ (m - 12)(m + 5) \]

Solve each equation. Check the solutions.

19. $x^2 + 17x + 42 = 0$
    \[ \{ -14, -3 \} \]

20. $p^2 + 5p - 84 = 0$
    \[ \{ -12, 7 \} \]

21. $h^2 + 3h - 54 = 0$
    \[ \{ -9, 6 \} \]

22. $n^2 - 12n + 64 = 0$
    \[ \{ -4, 16 \} \]

23. $n^2 + 4n - 32 = 0$
    \[ \{ -8, 4 \} \]

24. $h^2 + 17h - 60 = 0$
    \[ \{ -20, 3 \} \]

25. $x^2 - 56 = 0$
    \[ \{ -7, 7 \} \]

26. $x^2 + 4x - 12 = 0$
    \[ \{ -6, 2 \} \]

27. $x^2 + 8x - 54 = 0$
    \[ \{ -6, 9 \} \]

28. $y^2 + 2y = 0$
    \[ \{ -2, 0 \} \]

29. $v^2 + 16v + 26 = 0$
    \[ \{ -13, -1 \} \]

30. $x^2 + 3x - 35 = 0$
    \[ \{ -7, 5 \} \]

31. Find all values of $k$ so that the trinomial $x^2 + kx - 35$ can be factored using integers.
   \[ k = \{ -7, 5 \} \]

32. CONSTRUCTION A construction company is planning to pour concrete for a driveway.
   The length of the driveway is 16 feet longer than its width $w$.
   a. Write an expression for the area of the driveway.
   b. Find the dimensions of the driveway if it has an area of 260 square feet.

33. WEB DESIGN Janelle has a 10-inch by 12-inch photograph. She wants to scan the photograph, then reduce the result by the same amount in each dimension to post on her Web site. Janelle wants the area of the image to be one eighth that of the original photograph.
   a. Write an equation to represent the area of the reduced image.
   b. Find the reduced dimensions. 3 in. by 5 in. long
8-3 Enrichment

Puzzling Primes
A prime number has only two factors, itself and 1. The number 6 is not prime because it has 2 and 3 as factors; 5 and 7 are prime. The number 1 is not considered to be prime.

1. Use a calculator to help you find the 35 prime numbers less than 100.
2. Find the prime numbers generated by Euler’s formula for x from 0 through 7.
3. Show that the trinomial $x^2 + x + 31$ will not give prime numbers for very many values of x. It works for $x = 0, 2, 3, 5,$ and 6 but not for $x = 1, 4,$ and 7.
4. Find the largest prime number generated by Euler’s formula.
5. Goldbach’s Conjecture is that every nonzero even number greater than 2 can be written as the sum of two primes. No one has ever proved that this is always true, but no one has found a counterexample, either.
6. Show that Goldbach’s Conjecture is true for the first 5 even numbers greater than 2.

8-4 Study Guide and Intervention

Quadratic Equations: $ax^2 + bx + c = 0$

Factor $ax^2 + bx + c$. To factor a trinomial of the form $ax^2 + bx + c$, find two integers, $m$ and $p$ whose product is equal to $ac$ and whose sum is equal to $b$. If there are no integers that satisfy these requirements, the polynomial is called a prime polynomial.

Example 1

Factor $2x^2 + 15x + 18$.

In this example, $a = 2, b = 15,$ and $c = 18$.

You need to find two numbers that have a sum of 15 and a product of 2 • 18 or 36.

Make a list of the factors of 36 and look for the pair of factors with a sum of 15.

<table>
<thead>
<tr>
<th>Factors of 36</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 36</td>
<td>37</td>
</tr>
<tr>
<td>2, 18</td>
<td>20</td>
</tr>
<tr>
<td>3, 12</td>
<td>15</td>
</tr>
</tbody>
</table>

Use the pattern $ax^2 + mx + px + c$, with $a = 2, m = 3, p = 12,$ and $c = 18$.

$2x^2 + 15x + 18 = 2x^2 + 3x + 12x + 18$

$= (2x^2 + 3x) + (12x + 18)$

$= x(2x + 3) + 6(2x + 3)$

$= (x + 6)(2x + 3)$

Therefore, $2x^2 + 15x + 18 = (x + 6)(2x + 3)$.

Exercises

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write prime.

1. $2x^2 - 3x - 2$

2. $3m^2 - 8m - 3$

3. $16r^2 + 8r + 1$

4. $2x^2 + 3x - 2$

5. $5x^2 + 2x - 8$

6. $16r^2 - 25r - 5$

7. $2x^2 + 5a + 3$

8. $18a^2 + 9a - 5$

9. $-4t^2 + 19t - 21$

10. $8x^2 - 4x - 24$

11. $28x^2 + 60p - 25$

12. $48x^2 + 22x - 15$

13. $3y^2 - 6y - 24$

14. $4x^2 + 26x - 48$

15. $8y^2 - 44m + 48$

16. $6x^2 - 7x + 18$

17. $2a^2 - 14a + 18$

18. $18 + 11y + 2a^2$
8-4 Skills Practice

**Quadratic Equations: ax^2 + bx + c = 0**

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write prime.

1. \(2x^2 + 5x + 2\) prime
   \((x + 2)(2x + 1)\)

2. \(3x^3 + 5x + 2\) prime
   \((3n + 2)(n + 1)\)

3. \(2x^2 + 9x - 5\)
   \((t + 5)(2t - 1)\)

4. \(3x^2 - 7x + 2\)
   \((3g - 1)(g - 2)\)

5. \(2x^2 - 11x + 15\)
   \((t - 3)(2t - 5)\)

6. \(2x + 3x - 6\)

7. \(y^2 + y - 1\)
   \((y + 1)(2y - 1)\)

8. \(4h^2 + 8h - 5\)
   \((2h + 5)(2h - 1)\)

9. \(4x^2 - 3x - 3\)
   \((4b - 1)(6 + 4)\)

10. \(4x^2 + 6x - 8\)
   \((3p - 2)(3p + 4)\)

11. \(3x^2 + 3x + 63\)
   \((2w - 5)(5w + 3)\)

12. \(5y^2 - 13y + 10\)
   \((2w + 7)(2w - 3)\)

Solve each equation. Check the solutions.

13. \(x^2 + 3x = 6\)
   prime

14. \(y^2 + 6y - 12\)
   \((2w - 4)(3w + 2)\)

15. \(2x^2 + 3x = 2\)
   \(-3 - \frac{1}{2}\)

16. \(3x^2 + 2x - 3 = 0\)
   \(1, \frac{3}{2}\)

17. \(4x^2 - x - 3 = 0\)
   \(1, \frac{3}{2}\)

18. \(5x^2 + 2x - 3 = 0\)
   \(-1, \frac{3}{2}\)

19. \(6x^2 - 11x - 10 = 0\)
   \(-\frac{5}{3}, \frac{2}{3}\)

20. \(7x^2 + 3x + 2\)
   \(-\frac{4}{3}, \frac{2}{3}\)

21. \(8x^2 - 18x + 5 = 15\)
   \(-\frac{3}{2}, \frac{2}{3}\)

22. \(9x^2 - 23x + 12 = 0\)
   \(-\frac{2}{3}, \frac{4}{3}\)

23. \(10x^2 - 15x = 6x - 12\)
   \(-\frac{4}{3}, \frac{2}{3}\)

24. \(10x^2 - 15x = 6x - 12\)
   \(-\frac{4}{3}, \frac{2}{3}\)

25. \(10x^2 - 15x = 6x - 12\)
   \(-\frac{4}{3}, \frac{2}{3}\)
### Quadratic Equations: $ax^2 + bx + c = 0$  

**Practice**

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write prime.

<table>
<thead>
<tr>
<th>$2b^2 + 10b + 12$</th>
<th>$2y^2 + 8y + 4$</th>
<th>$3y^2 + 4y - 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2(b + 2)(b + 3)$</td>
<td>$(y + 2)(y + 2)$</td>
<td>$(y + 1)(y - 3)$</td>
</tr>
<tr>
<td>$2y^2 - 5y - 10$</td>
<td>$3m^2 + 2m - 3$</td>
<td>$2y^2 - 17y + 12$</td>
</tr>
<tr>
<td>prime</td>
<td>$(3m - 1)(2m + 3)$</td>
<td>$(4w - 3)(2w - 3)$</td>
</tr>
<tr>
<td>$7x^2 - 17x + 12$</td>
<td>$8x^2 - 18x + 9$</td>
<td>$10.5x^2 - n - 28$</td>
</tr>
<tr>
<td>prime</td>
<td>$(3a - 4)(2a - 3)$</td>
<td>$(5n - 7)(3n + 4)$</td>
</tr>
</tbody>
</table>

**Word Problem Practice**

### Quadratic Equations: $ax^2 + bx + c = 0$

1. **BREAK EVEN**  
   Breaking even occurs when the revenues for a business equal the cost. A local children's museum studied their costs (wages, electricity, etc.) and revenues from paid admission. They found that their break-even time is given by the equation $2b^2 - 2b - 24 = 0$, where $b$ is the number of hours the museum is open per day. How many hours must the museum be open per day to reach the break even point?  
   **Hint:** Use the Pythagorean Theorem to solve the problem.  
   **4 hours**

2. **CARPENTRY**  
   Miko wants to build a toy box for her sister. It is to be 2 feet high, and the width is to be 3 feet less than its length. If it needs to hold a volume of 80 cubic feet, find the length and width of the box.

   - **length** = 8 ft; **width** = 5 ft

3. **FURNITURE**  
   The student council wants to purchase a table for the school lobby. The table comes in a variety of dimensions, but for every table, the length is 1 meter greater than twice the width. The student council has budgeted for a table top with an area of exactly 3 square meters.

   a. Write a quadratic equation (set equal to zero) to represent the information.
   b. Using 3 as an approximation for $\pi$, solve the equation for $r$. \{5, -32\}
   c. What radius should the student council budget for in order to purchase a table that is exactly 3 square meters?  
   
   Find the width and length of the table they can purchase.  
   - **width** = 1 m; **length** = 3 m

4. **LADDERS**  
   A ladder is resting against a wall. The top of the ladder touches the wall at a height of 15 feet, and the length of the ladder is one foot more than twice its distance from the wall. Find the distance from the wall to the bottom of the ladder. (**Hint:** Use the Pythagorean Theorem to solve the problem.)  
   **8 ft**

5. **FARMING**  
   Mr. Hensley has a total of 480 square feet of sheet metal with which he would like to construct a cylindrical tank for storing grain. The local zoning law limits the height of the tank to 13.5 feet. Recall that a formula for the surface area of a bottomless cylinder with radius $r$ and height $h$ is $A = \pi r^2 + 2\pi rh$.

   a. Write a quadratic equation (set equal to zero) to represent the information.
   b. Using 3 as an approximation for $\pi$, solve the equation for $r$. \{5, -32\}
   c. What radius should Mr. Hensley use for his tank?  
   **5 ft**

### LADDERS

![Ladder Diagram](image-url)

- Wall
- Ladder
- Right Angle

### Solutions

- **1.** $(a - 4)(2a - 3)$
- **2.** $(y + 2)(y + 2)$
- **3.** $(y + 1)(y - 3)$
- **4.** $(3m - 1)(2m + 3)$
- **5.** $(4w - 3)(2w - 3)$
- **6.** $(5n - 7)(3n + 4)$

**Answers**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>19. $3h^2 + 2h - 16 = 0$</td>
<td><strong>0, -2/3</strong></td>
</tr>
<tr>
<td>20. $15n^2 - n = 2$</td>
<td><strong>1, 3/5</strong></td>
</tr>
<tr>
<td>21. $8q^2 - 10q + 3 = 0$</td>
<td><strong>-1/2, 1/4</strong></td>
</tr>
<tr>
<td>22. $6b^2 + 9b + 4$</td>
<td><strong>2, -1/3</strong></td>
</tr>
<tr>
<td>23. $10r^2 - 21r - 4r + 6$</td>
<td><strong>-1, 9/2</strong></td>
</tr>
<tr>
<td>24. $10g^2 + 10 = 29g$</td>
<td><strong>1, 5</strong></td>
</tr>
<tr>
<td>25. $6y^2 - 7y - 2$</td>
<td><strong>-1, 2/3</strong></td>
</tr>
<tr>
<td>26. $9r^2 = -6z + 15$</td>
<td><strong>-5, 3</strong></td>
</tr>
<tr>
<td>27. $12h^2 + 15 = 16h + 20$</td>
<td><strong>-2, 3/2</strong></td>
</tr>
<tr>
<td>28. $2x^2 - 3x - 1 = 0$</td>
<td><strong>-1, 1/2</strong></td>
</tr>
<tr>
<td>29. $8y^2 - 16y + 6y - 12$</td>
<td><strong>2, -1/3</strong></td>
</tr>
<tr>
<td>30. $18r^2 + 10r - 11a + 4$</td>
<td><strong>2, -1/3</strong></td>
</tr>
</tbody>
</table>
**Graphing Calculator Activity**

**Using Tables in Factoring by Grouping**

The **TABLE** feature can be used to help factor a polynomial by finding the factors of a certain product, which have a specific sum.

**Example 1**

Factor \(10x^2-43x+28\) by grouping.

To draw a rectangular model, the value 2 was used for \(x\) so that the shorter side would have a length of 1. Then the drawing was done in centimeters. So, the area of the rectangle is \(x^2+5x-6\).

To draw a right triangle model, recall that the area of a triangle is one-half the base times the height. So, one of the sides must be twice as long as the shorter side of the rectangular model.

\[
x^2+5x-6 = (x-1)(x+6)
\]

The area of the right triangle is also \(x^2+5x-6\).

**Factor each trinomial. Then follow the directions to draw each model of the trinomial.**

1. \(x^2+2x-3\) Use \(x = 2\). Draw a rectangle in centimeters. \((x+3)(x-1)\)

2. \(3x^2+5x-2\) Use \(x = 1\). Draw a rectangle in centimeters. \((x+2)(3x-1)\)

3. \(x^2-4x+3\) Use \(x = 4\). Draw two different right triangles in centimeters. \((x-1)(x-3)\)

4. \(9x^2-9x+2\) Use \(x = 2\). Draw two different right triangles. Use 0.5 centimeter for each unit. \((3x-2)(3x-1)\)

**Exercises**

Factor each quadratic polynomial if possible.

1. \(x^2+2x-96\) \((y+32)(y-3)\)

2. \(x^2-14x+51\) \((x-17)(x+3)\)

3. \(3x^2+16x-35\) \((3x-5)(x+7)\)

4. \(4x^2-25x+18\) \(5,6a^2-a-15\) \((3a-5)(2a+3)\)

5. \(6x^2+13x+6\) \((2m+3)(3m+2)\)

6. \(7x^2-x-6\) \((4-3)(3x+2)\)

7. \(8y^2+49y+25\) \((4y+5)^2\)

8. \(9y^2+24y-493\) \((2y+29)(2b-17)\)
8-5 Study Guide and Intervention

Chapter 8

NAME ________________________ DATE ______ PERIOD ______

8-5 Study Guide and Intervention

Chapter 8

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8-5 Study Guide and Intervention

Chapter 8
# 8-5 Skills Practice

## Quadratic Equations: Differences of Squares

Factor each polynomial, if possible. If the polynomial cannot be factored, write prime.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $x^2 - 4$</td>
<td>2. $n^2 - 64$</td>
</tr>
<tr>
<td>$(a + 2)(a - 2)$</td>
<td>$(n + 8)(n - 8)$</td>
</tr>
<tr>
<td>3. $1 - 49d^2$</td>
<td>4. $-16 + p^2$</td>
</tr>
<tr>
<td>$(1 + 7d)(1 - 7d)$</td>
<td>$(p + 4)(p - 4)$</td>
</tr>
<tr>
<td>5. $k^2 + 25$</td>
<td>6. $36 - 100u^2$</td>
</tr>
<tr>
<td>prime</td>
<td>$(6 - 10w)(6 + 10w)$</td>
</tr>
<tr>
<td>7. $r^2 - 81u^2$</td>
<td>8. $4h^2 - 25g^2$</td>
</tr>
<tr>
<td>$(f + 9u)(f - 9u)$</td>
<td>$(2h + 5g)(2h - 5g)$</td>
</tr>
<tr>
<td>9. $64m^2 - 9y^2$</td>
<td>10. $4c^2 - 5d^2$</td>
</tr>
<tr>
<td>$(8m - 3y)(8m + 3y)$</td>
<td>prime</td>
</tr>
</tbody>
</table>

### Solve each equation by factoring. Check your solutions.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>11. $-49b^2 + 4f^2$</td>
<td>12. $8k^2 - 72p^2$</td>
</tr>
<tr>
<td>$(2t + 7r)(2t - 7r)$</td>
<td>$8(k + 3p)(k - 3p)$</td>
</tr>
<tr>
<td>13. $20q^2 - 5r^2$</td>
<td>14. $32a^2 - 50b^2$</td>
</tr>
<tr>
<td>$5(2q + r)(2q - r)$</td>
<td>$2(4a + 5b)(4a - 5b)$</td>
</tr>
</tbody>
</table>

# 8-5 Practice

## Quadratic Equations: Differences of Squares

Factor each polynomial, if possible. If the polynomial cannot be factored, write prime.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $k^2 + 100$</td>
<td>2. $81 - r^2$</td>
</tr>
<tr>
<td>$(k + 10)(k - 10)$</td>
<td>$(9 + r)(9 - r)$</td>
</tr>
<tr>
<td>4. $4x^2 + 25$</td>
<td>5. $144 - 9t^2$</td>
</tr>
<tr>
<td>prime</td>
<td>$(12 + 3f)(12 - 3f)$</td>
</tr>
<tr>
<td>6. $36p^2 - 49h^2$</td>
<td>prime</td>
</tr>
<tr>
<td>$(6g + 7h)(6g - 7h)$</td>
<td></td>
</tr>
<tr>
<td>7. $121m^2 - 144p^2$</td>
<td>8. $32 - 8y^2$</td>
</tr>
<tr>
<td>$(11m - 12p)(11m + 12p)$</td>
<td>$8(2 - y)(2 + y)$</td>
</tr>
<tr>
<td>9. $28c^2 - 54b^2$</td>
<td>10. $32t^2 - 18u^2$</td>
</tr>
<tr>
<td>$6(2a - 3b)(2a + 3b)$</td>
<td>prime</td>
</tr>
<tr>
<td>$2(4t - 3u)(4t + 3u)$</td>
<td></td>
</tr>
</tbody>
</table>

### Solve each equation by factoring. Check your solutions.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>11. $-49b^2 + 4f^2$</td>
<td>12. $8k^2 - 72p^2$</td>
</tr>
<tr>
<td>$(2t + 7r)(2t - 7r)$</td>
<td>$8(k + 3p)(k - 3p)$</td>
</tr>
<tr>
<td>13. $20q^2 - 5r^2$</td>
<td>14. $32a^2 - 50b^2$</td>
</tr>
<tr>
<td>$5(2q + r)(2q - r)$</td>
<td>$2(4a + 5b)(4a - 5b)$</td>
</tr>
</tbody>
</table>

## Erosion

A rock drops from a cliff and plunges toward the ground 400 feet below. The distance that the rock falls in $t$ seconds is given by the equation $d = 16t^2$. How long does it take the rock to hit the ground? $\frac{5}{8}$ seconds.

## Forensics

Mr. Cooper contested a speeding ticket given to him after he applied his brakes and skidded to a halt to avoid hitting another car. In traffic court, he argued that the length of the skid marks on the pavement, 150 feet, proved that he was driving under the posted speed limit of 65 miles per hour. The ticket cited his speed at 70 miles per hour. Use the formula $d = \frac{1}{2}at^2$, where $a$ is the speed of the car and $t$ is the length of the skid marks, to determine Mr. Cooper's speed when he applied the brakes. Was Mr. Cooper correct in claiming that he was not speeding when he applied the brakes? 60 mph; yes.
8-5 Word Problem Practice

Quadratic Equations: Differences of Squares

1. LOTTERY A state lottery commission analyzes the ticket purchasing patterns of its citizens. The following expression is developed to help officials calculate the likely number of people who will buy tickets for a certain size jackpot.

\[ 81x^2 - 36y^2 \]

Find the expression completely.

\[ 3(3a + 2b)(3a - 2b) \]

2. OPTICS A reflector on the inside of a certain flashlight is a parabola given by the equation \( y = x^2 - 25 \). Find the points where the reflector meets the lens by finding the values of \( x \) when \( y = 0 \).

\[ 5, -5 \]

3. ARCHITECTURE The drawing shows a triangular roof truss with a base measuring the same as its height. The area of the truss is 98 square meters.

Find the height of the truss. 14 m

4. BALLOONING The function \( f(t) = -16t^2 + 576 \) represents the height of a freely falling ballast bag that starts from rest on a balloon 576 feet above the ground. After how many seconds \( t \) does the ballast bag hit the ground?

After 6 seconds

5. DECORATING Marvin wants to purchase a rectangular rug. It has an area of 80 square feet. He cannot remember the length and width, but he remembers that the length was 8 more than some number and the width was 8 less than that same number.

a. Write a quadratic equation using the information given. \( x^2 - 64 = 80 \) or \( x^2 - 144 = 0 \)

b. What are the length and width of the rug? 20 ft and 4 ft

8-5 Enrichment

Factoring Trinomials of Fourth Degree

Some trinomials of the form \( a^4 + a^2b^2 + b^4 \) can be written as the difference of two squares and then factored.

Example

Factor \( 4x^4 - 3x^2y^2 + 9y^4 \).

Step 1 Find the square roots of the first and last terms.

\[ \sqrt{4x^4} = 2x^2 \]

\[ \sqrt{9y^4} = 3y^2 \]

Step 2 Find twice the product of the square roots.

\[ 2(2x^2y)(3y^2) = 12x^2y^2 \]

Step 3 Separate the middle term into two parts. One part is either your answer to Step 2 or its opposite. The other part should be the opposite of a perfect square.

\[ -3x^2y^2 = -12x^2y^2 - 25x^2y^2 \]

Step 4 Rewrite the trinomial as the difference of two squares and then factor.

\[ 4x^4 - 37x^2y^2 + 9y^4 = (4x^4 - 12x^2y^2 + 9y^4) - 25x^4y^4 \]

\[ = (2x^2 - 3y^2)^2 - 25x^2y^4 \]

\[ = (2x^2 - 3y^2)^2 + 5xy(2x^2 - 3y^2) - 5xy \]

\[ = (2x^2 + 5xy - 3y^2)(2x^2 - 5xy - 3y^2) \]

Factor each polynomial.

1. \( x^4 + x^2y^2 + y^4 \)

   \[ x^4 + x^2 + 1 \]

2. \( x^4 + x^2 \)

   \[ (x^2 + x + 1)(x^2 - x + 1) \]

3. \( 9a^4 - 15a^2 + 1 \)

   \[ (3a^2 + 3a - 1)(3a^2 - 3a - 1) \]

4. \( 16a^4 - 17a^2 + 1 \)

   \[ (4a - 1)(a + 1)(4a + 1)(a - 1) \]

5. \( 4a^4 - 13a^2 + 1 \)

   \[ (2a^2 + 3a - 1)(2a^2 - 3a - 1) \]

6. \( 9a^4 + 26a^2b^2 + 25b^4 \)

   \[ (3a^2 + 2ab + 5b^2)(3a^2 - 2ab + 5b^2) \]

7. \( 4a^4 - 21a^2y^2 + 9y^4 \)

   \[ (2x^2 + 3xy - 3y^2)(2x^2 - 3xy - 3y^2) \]

8. \( 4a^4 - 29a^2b^2 + 25b^4 \)

   \[ (2a + 5b)(a - b)(2a - 5b)(a + b) \]
**8-5 Spreadsheet Activity**

**Differences of Squares**

There is a special pattern you can use to factor binomials of the form $a^2 - b^2$. You can use a spreadsheet to discover this relationship.

**Example**  Use a spreadsheet to investigate the values of the expressions $(a^2 - b^2)$, $(a - b)(a + b)$, and $(a + b)^2$. What conjecture can you make about the expressions?

**Step 1** You will use Columns A and B to enter various values that you choose for $a$ and $b$.

**Step 2** Enter the formulas for $(a^2 - b^2)$, $(a - b)(a + b)$, and $(a + b)^2$ in Columns C, D, E, and F. To enter an exponent, use the symbol $^2$ followed by the exponent. For example, the square of the value in cell A2 is entered as A2$^2$.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a</td>
<td>$a^2$</td>
<td>$a^2$</td>
<td>$a^2$</td>
<td>$a^2$</td>
</tr>
<tr>
<td>2</td>
<td>b</td>
<td>$b^2$</td>
<td>$a^2$</td>
<td>$a^2$</td>
<td>$a^2$</td>
</tr>
<tr>
<td>3</td>
<td>$a^2 - b^2$</td>
<td>$(a - b)(a + b)$</td>
<td>$(a + b)^2$</td>
<td>$(a + b)^2$</td>
<td>$(a + b)^2$</td>
</tr>
<tr>
<td>4</td>
<td>$a^2 + b^2$</td>
<td>$a^2 + b^2$</td>
<td>$a^2 + b^2$</td>
<td>$a^2 + b^2$</td>
<td>$a^2 + b^2$</td>
</tr>
</tbody>
</table>

**Exercises**

1. Enter various values of $a$ and $b$ in Columns A and B. Look for a pattern. What do you observe about the expressions? For any values of $a$ and $b$, $(a^2 - b^2) = (a - b)(a + b)$.

2. Find the products of $(a - b)^2$, $(a - b)(a + b)$, and $(a + b)^2$. Do the results verify your conjecture? $(a - b)^2 = a^2 - 2ab + b^2$; $(a - b)(a + b) = a^2 - b^2$; and $(a + b)^2 = a^2 + 2ab + b^2$.

3. Use the pattern you observed to factor each binomial.

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>$(m^2 - n^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>$9 - 4m + n$</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>$16 - 4m + n$</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>$25 - 8m + n$</td>
</tr>
</tbody>
</table>

4. Use a spreadsheet to factor perfect square trinomials.

5. Use a spreadsheet to factor perfect square trinomials.

**8-6 Study Guide and Intervention**

**Quadratic Equations: Perfect Squares**

**Factor Perfect Square Trinomials**

A trinomial of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$

The patterns shown below can be used to factor perfect square trinomials.

<table>
<thead>
<tr>
<th>Squaring a Binomial</th>
<th>Factoring a Perfect Square Trinomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(a + b)^2 = a^2 + 2ab + b^2$</td>
<td>$(a + b)(a + b)$</td>
</tr>
<tr>
<td>$(a - b)^2 = a^2 - 2ab + b^2$</td>
<td>$(a - b)(a - b)$</td>
</tr>
</tbody>
</table>

**Example 1** Determine whether $16a^2 - 24a + 9$ is a perfect square trinomial. If so, factor it.

Since $16a^2 = (4a)^2$, the first term is a perfect square.

Since $9 = 3^2$, the last term is a perfect square.

The middle term is equal to $2(4a)(3)$. Therefore, $16a^2 - 24a + 9$ is a perfect square trinomial.

$16a^2 - 24a + 9 = (4a - 3)^2$

**Example 2** Factor $16a^2 - 32a + 15$.

Since $9$ is not a perfect square, use a different factoring pattern.

$16a^2 - 32a + 15 = (4a)^2 - 2(4a)(5) + 5^2$;

Write the pattern.

Therefore, the GCF is $5$; factor by grouping.

Therefore $16a^2 - 32a + 15 = 4a(4a - 5) + 5(4a - 5)$.

**Exercises**

Determine whether each trinomial is a perfect square trinomial. Write yes or no.

If so, factor it.

1. $t^2 - 16t + 64$; yes; $(t - 8)(t - 8)$
2. $m^2 + 10m + 25$; yes; $(m + 5)(m + 5)$
3. $p^2 - 8p + 64$; no

Factor each polynomial, if possible. If the polynomial cannot be factored, write prime.

4. $8t^2 - 200y^2$; prime
5. $5x^2 + 22x + 121$; $(5x + 11)^2$
6. $81 + 18j + j^2$; $(9 + j)^2$
7. $25c^2 - 10c - 1$; prime
8. $169 - 26r + r^2$; $(13 - r)^2$
9. $9x^2 - 9x + 2$; prime
10. $14x^2 + 49m + 36$; $(7x + 3)^2$
11. $16x^2 - 25am^2$; $(4a - 5a)(4a - 5a)$
12. $16b^2 - 16b + 256$; prime
13. $36x^2 - 12b + 1$; prime
14. $16x^2 - 40bx + 25b^2$; $(4a - 5b)^2$
15. $8m^2 - 64m$; $8(m^2 - 8)$
**8-6 Study Guide and Intervention (continued)**

**Quadratic Equations: Perfect Squares**

Solve Equations with Perfect Squares

Factoring and the Zero Product Property can be used to solve equations that involve repeated factors. The repeated factor gives just one solution to the equation. You may also be able to use the square root property below to solve certain equations.

<table>
<thead>
<tr>
<th>Square Root Property</th>
<th>For any number $n &gt; 0$, if $x^n = a$, then $x = \pm \sqrt[n]{a}$.</th>
</tr>
</thead>
</table>

**Example**

Solve each equation. Check your solutions.

a. $x^2 - 6x + 9 = 0$

Original equation

$x^2 - 2(3)x + 3^2 = 0$

$x - 3 = 0$

$x = 3$

The solution set is {3}. Since $3^2 - 6(3) + 9 = 0$, the solution checks.

b. $(a - 5)^2 = 64$

Original equation

$a - 5 = \pm \sqrt{64}$

$a - 5 = 8$ or $a - 5 = -8$

Add 5 to each side.

$a = 5 + 8$ or $a = 5 - 8$

Separate into two equations.

$a = 13$ or $a = -3$

Solve each equation.

The solution set is $\{-3, 13\}$. Since $-3 - 5)^2 = 64$ and $(13 - 5)^2 = 64$, the solutions check.

**Exercises**

Solve each equation. Check your solutions.

1. $x^2 + 4x + 4 = 0 \{4\}$

2. $16x^2 + 16x + 4 = 0 \{-1\}$

3. $25x^2 - 10d + 1 = 0 \{\frac{1}{5}\}$

4. $x^2 + 10x + 25 = 0 \{-5\}$

5. $9x^2 - 6x + 1 = 0 \{\frac{1}{3}\}$

6. $x^2 + x + \frac{1}{4} = 0 \{-\frac{1}{2}\}$

7. $25x^2 + 20k + 4 = 0 \{-\frac{2}{5}\}$

8. $p^2 + 2p + 1 = 49$

9. $x^2 + 4x + 4 = 64$

-8, 6

-10, 6

10. $x^2 - 6x + 9 = 25 \{-2, 6\}$

11. $a^2 + 5a + 16 = 1 \{-3, -5\}$

12. $16y^2 + 8y + 1 = 0 \{-\frac{1}{4}\}$

13. $(x + 3)^2 = 49 \{-10, 4\}$

14. $(y + 6)^2 = 1 \{-7, -5\}$

15. $(m - 7)^2 = 49 \{0, 14\}$

16. $(x + 1)^2 = 1 \{-1, 0\}$

17. $(4x + 3)^2 = 25 \{2, -2\}$

18. $(3x - 2)^2 = 4 \{\frac{1}{3}, 1\}$

19. $(x + 1)^2 = 7 \{x = \pm \sqrt{7}\}$

20. $(y - 3)^2 = 6 \{3 \pm \sqrt{6}\}$

21. $(m - 2)^2 = 5 \{2 \pm \sqrt{5}\}$

**8-6 Skills Practice**

**Quadratic Equations: Perfect Squares**

Determine whether each trinomial is a perfect square trinomial. Write yes or no.

If no, factor it.

1. $m^2 - 6m + 9$

2. $r^2 + 4r + 4$

3. $a^2 - 14a + 49$

4. $2w^2 - 4w + 9$

5. $4d^2 - 4d + 1$

6. $9b^2 + 30b + 25$

Factor each polynomial, if possible. If the polynomial cannot be factored, write prime.

7. $2x^2 - 72$

8. $6b^2 + 11b + 3$

9. $36c^2 - 24c + 4$

10. $16h^2 - 56$

11. $17e^2 - 24eb$

12. $q^2 - 14q + 36$

13. $9y^2 + 24y + 144$

14. $6d^2 - 96$

15. $x^2 - 18x + 81 = 0 \{9\}$

16. $4p^2 + 4p + 1 = 0 \{-\frac{1}{2}\}$

17. $9g^2 - 12g + 4 = 0 \{\frac{2}{3}\}$

18. $y^2 - 16y + 64 = 81 \{-1, 17\}$

19. $4n^2 - 17 = 0 \{\pm 3\}$

20. $x^2 + 30x + 150 = -75 \{-15\}$

21. $(k + 2)^2 = 16 \{-6, 2\}$

22. $(m - 4)^2 = 7 \{2 \pm \sqrt{7}\}$
8-6 Practice
Quadratic Equations: Perfect Squares

Determine whether each trinomial is a perfect square trinomial. Write yes or no. If so, factor it.

1. \(m^2 + 16m + 64\)
   - yes; \((m + 8)^2\)

2. \(9x^2 - 6x + 1\)
   - yes; \((3x - 1)^2\)

3. \(4x^2 - 20x + 25\)
   - yes; \((2x - 5)^2\)

4. \(16p^2 + 24p + 9\)
   - yes; \((4p + 3)^2\)

5. \(25b^2 - 45b + 16\)
   - no

6. \(49k^2 - 56k + 16\)
   - yes; \((7k - 4)^2\)

Factor each polynomial, if possible. If the polynomial cannot be factored, write prime.

7. \(3p^2 - 147\)
   - \(3(p + 7)(p - 7)\)

8. \(6x^2 + 11x - 35\)
   - \((2x + 7)(3x - 5)\)

9. \(9.50p^2 - 60p + 18\)
   - \(2(5q^2 - 3q)^2\)

10. \(6r^2 - 14r - 12\)
    - \((2r - 1)(3r - 6)\)

11. \(11.6r^2 - 18\)
    - \(12.30r^2 - 30r + 12\)

12. \(2(3t^2 + 2t - 3)\)
    - \(2(5k + 3)(3k + 2)\)

13. \(15r^2 - 24r\)
    - \(15.9r^2 - 30r - 25\)

14. \(3b(5b - 8f)\)
    - prime

15. \(12h^2 - 60h + 75\)
    - \(2(5h - 3)(3h + 5)\)

16. \(7u^2 - 28u\)
    - \((u - 2m)(u + 2m)\)

17. \(w^2 + 1)(w + 3)(w - 3)\)
    - \(4a + 9d)^2\)

Solve each equation. Check the solutions.

18. \(4k^2 - 28k = -49\)
   - \(\left\{\frac{1}{2}, 7\right\}\)

19. \(50k^2 + 20k + 2 = 0\)
   - \(\left\{\frac{1}{2} \pm 1\right\}\)

20. \(22k^2 + \frac{7}{9}k + \frac{1}{9} = 0\)
   - \(\left\{-\frac{7}{9} \pm \frac{1}{3}\right\}\)

21. \(x^2 = 12x + 36 = 25\)
   - \(\{1, 5\}\)

22. \(y^2 - 8y + 16 = 64\)
   - \(\{4, 12\}\)

23. \(a^2 + 9f^2 = 3\)
   - \(\{3 \pm \sqrt{3}\}\)

24. \(27w^2 - 6w + 9 = 13\)
   - \(\{-9 \pm \sqrt{3}\}\)

28. GEOMETRY The area of a circle is given by the formula \(A = \pi r^2\), where \(r\) is the radius. If the radius of a circle by 1 inch increases the resulting circle area of 100\(\pi\) square inches, what is the radius of the original circle? 9 in.

29. PICTURE FRAMING Mikaela placed a frame around a print that measures 10 inches by 10 inches. The area of just the frame itself is 99 square inches. What is the width of the frame? 1.5 in.

8-6 Word Problem Practice
Quadratic Equations: Perfect Squares

1. CONSTRUCTION The area of Liberty Township's square playground is represented by the trinomial \(x^2 - 10x + 25\). Write an expression using the variable \(x\) that represents the perimeter. \(4x - 20\) or \(4(x - 5)\)

2. AMUSEMENT PARKS Funtown
   - Downtown wants to build a vertical motion ride where the passengers are launched straight upward from ground level with an initial velocity of 96 feet per second. The ride car's height \(h\) in feet after \(t\) seconds is \(h = 96t - 16t^2\). How many seconds after launch would the car reach 144 feet?
   - \(3\) seconds

3. BUSINESS Saini Sprinkler Company installs irrigation systems. To track monthly costs \(C\) and revenues \(R\), they use the following functions, where \(x\) is the number of systems they install.
   - \(R(x) = x^2 + 12x + 36\) and \(C(x) = 7x^2 + 20x - 12\)
   - The monthly profit can be found by subtracting cost from revenue.
   - \(R(x) - C(x)\)
   - Find a function to project monthly profit and use it to find the break-even point where the profit is zero.
   - \(P(x) = x^2 - 8x + 16; x = 4\)

4. GEOMETRY Holly can make an open-topped box out of a square piece of cardboard by cutting 3-inch squares from the corners and folding up the sides to meet. The volume of the resulting box is \(V = 3x^2 - 36x + 108\), where \(x\) is the original length and width of the cardboard.
   - a. Factor the polynomial expression from the volume equation. \(3(x - 6)(x - 6)\)
   - b. What is the volume of the box if the original length of each side of the cardboard was 14 inches? 192 in
   - c. What is the original side length of the cardboard when the volume of the box is 27 in? 9 in.
ERROR: undefined
OFFENDING COMMAND:
STACK: