### Anticipation Guide

**Solving Systems of Linear Equations**

#### Step 1  
**Before you begin Chapter 6**
- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

#### Step 2  
**After you complete Chapter 6**
- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

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### Study Guide and Intervention

**Graphing Systems of Equations**

**Possible Number of Solutions**

Two or more linear equations involving the same variables form a system of equations. A solution of the system of equations is an ordered pair of numbers that satisfies both equations. The table below summarizes information about systems of linear equations.

<table>
<thead>
<tr>
<th>Graph of a System</th>
<th>intersecting lines</th>
<th>same line</th>
<th>parallel lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Solutions</td>
<td>exactly one solution</td>
<td>infinitely many solutions</td>
<td>no solution</td>
</tr>
</tbody>
</table>

**Terminology**

<table>
<thead>
<tr>
<th>independent</th>
<th>dependent</th>
<th>consistent</th>
<th>inconsistent</th>
</tr>
</thead>
</table>

**Example**

Use the graph at the right to determine whether each system is consistent or inconsistent and if it is independent or dependent.

- **a.** \[ y = -x + 2 \]
  - Since the graphs of \( y = -x + 2 \) and \( y = x + 1 \) intersect, there is one solution. Therefore, the system is consistent and independent.

- **b.** \[ 3x + 3y = -3 \]
  - Since the graphs of \( y = -x + 2 \) and \( 3x + 3y = -3 \) are parallel, there are no solutions. Therefore, the system is inconsistent.

<table>
<thead>
<tr>
<th>Exercises</th>
<th>Use the graph at the right to determine whether each system is consistent or inconsistent and if it is independent or dependent.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. [ y = -x - 3 ]</td>
<td>consistent; independent</td>
</tr>
<tr>
<td>2. [ 2x + 2y = -6 ]</td>
<td>consistent; dependent</td>
</tr>
<tr>
<td>3. [ y = -x - 3 ]</td>
<td>inconsistent</td>
</tr>
<tr>
<td>4. [ 2x + 2y = -6 ]</td>
<td>consistent; independent</td>
</tr>
</tbody>
</table>

---

Answers

**Lesson 6-1**

**Possible Number of Solutions**

- A system of equations of parallel lines will have no solutions.
- A system of equations of two perpendicular lines will have infinitely many solutions.

**Example**

- \[ y = x + 2 \]
  - Since the graphs of \( y = x + 2 \) and \( y = x + 1 \) intersect, there is one solution. Therefore, the system is consistent and independent.

- \[ 3x + 3y = -3 \]
  - Since the graphs of \( y = -x + 2 \) and \( 3x + 3y = -3 \) are parallel, there are no solutions. Therefore, the system is inconsistent.

**Exercises**

- \[ y = -x - 3 \]
  - consistent; independent

- \[ 2x + 2y = -6 \]
  - consistent; dependent

- \[ 3x + y = 3 \]
  - inconsistent; dependent
Study Guide and Intervention

Graphing Systems of Equations

Solve by Graphing One method of solving a system of equations is to graph the equations on the same coordinate plane.

**Example**

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

a. \( x + y = 2 \)
\( x - y = 4 \)

The graphs intersect. Therefore, there is one solution. The point \((3, -1)\) seems to lie on both lines. Check this estimate by replacing \(x\) with 3 and \(y\) with \(-1\) in each equation.

\[
\begin{align*}
3 + (-1) & = 2 \quad \text{true} \\
3 - (-1) & = 4 \quad \text{true}
\end{align*}
\]

The solution is \((3, -1)\).

b. \( y = 2x + 1 \)
\( 2y = 4x + 2 \)

The graphs coincide. Therefore there are infinitely many solutions.

**Exercises**

Graph each system and determine the number of solutions it has. If it has one solution, name it.

1. \( y = -2 \) one; \((-1, -2)\)

2. \( x = 2 \) one; \((2, -3)\)

3. \( y = \frac{1}{2}x \) one; \((2, 1)\)

4. \( 2x + y = 6 \) one; \((1, 4)\)

5. \( 3x + 2y = 6 \) no solution

6. \( 2y = -4x + 4 \) infinitely many

7. \( x - y = 3 \)

8. \( y = x + 2 \) infinitely many

9. \( x + 3y = -3 \) one; \((-3, 0)\)

10. \( y - x = -1 \) one; \((1, 1)\)

11. \( x - y = 3 \) one; \((3, 0)\)

12. \( x + 2y = 4 \) infinitely many

13. \( y = 2x + 3 \) no solution
Chapter 6

Graphing Systems of Equations

Practice

NAME __________  DATE __________  PERIOD __________

6-1

Word Problem Practice

Glencoe Algebra 1

Graphing Systems of Equations

1. BUSINESS The widget factory will sell a total of y widgets after x days according to the equation \( y = 20x + 300 \). The factory will sell y gadgets after x days according to the equation \( y = 200x + 100 \). Look at the graph of the system of equations and determine whether it has no solution, one solution, or infinitely many solutions.

2. ARCHITECTURE An office building has two elevators. One elevator starts out on the 4th floor, 35 feet above the ground, and is descending at a rate of 2.2 feet per second. The other elevator starts out at ground level and is rising at a rate of 1.7 feet per second. Write a system of equations to represent the situation.

Sample answer:
\( y = 35 - 2.2x; y = 1.7x \)

3. FITNESS Olivia and her brother William had a bicycle race. Olivia rode at a speed of 20 feet per second while William rode at a speed of 15 feet per second. Write a system of equations to represent the situation.

4. AVIATION Two planes are in flight near a local airport. One plane is at an altitude of 1000 meters and is ascending at a rate of 400 meters per minute. The second plane is at an altitude of 5900 meters and is descending at a rate of 300 meters per minute.

a. Write a system of equations that represents the progress of each plane.

\[ y = 400x + 1000; \quad y = 5900 - 300x \]

b. Make a graph that represents the progress of each plane.

b. Name the solution.

9. SALES A used book store also started selling used CDs and videos. In the first week, the store sold 40 used CDs and videos, at $4.00 per CD and $6.00 per video. The sales for both CDs and videos totaled $180.00.

a. Write a system of equations to represent the situation.

\[ 4c + 6v = 180 \]

b. Graph the system of equations.

c. How many CDs and videos did the store sell in the first week?

30 CDs and 10 videos

8. BUSINESS Nick plans to start a home-based business producing and selling gourmet dog treats. He figures it will cost $20 in operating costs per week plus $0.50 to produce each treat. He plans to sell each treat for $1.50.

a. Graph the system of equations \( y = 0.5x + 20 \) and \( y = 1.5x \) to represent the situation.

b. How many treats does Nick need to sell per week to break even?

20

7. \( x + 3y = 3 \) one; \( 3x - y = -3 \) infinitely many

6. \( y = 2x - 3 \) infinitely many

5. No solution

4. \( x + 3y = 3 \) inconsistent and dependent

3. \( x + y = -3 \) inconsistent and dependent

2. \( 2x - y = -3 \) inconsistent and dependent

1. \( x + y = 3 \)

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**6-1 Enrichment**

### Graphing a Trip

The distance formula, \( d = rt \), is used to solve many types of problems. If you graph an equation such as \( d = 50 \), the graph is a model for a car going at 50 mi/h. The time the car travels is \( t \); the distance in miles the car covers is \( d \). The slope of the line is the speed.

Suppose you drive to a nearby town and return. You average 50 mi/h on the trip out but only 25 mi/h on the trip home. The round trip takes 5 hours. How far away is the town?

The graph at the right represents your trip. Notice that the return trip is shown with a negative slope because you are driving in the opposite direction.

### Solve each problem.

1. **Estimate the answer to the problem in the above example. About how far away is the town?**
   - about 80 miles

2. **An airplane has enough fuel for 3 hours of safe flying. On the trip out, the pilot averages 200 mi/h flying against a headwind. On the trip back, the pilot averages 250 mi/h. How long a trip out can the pilot make?**
   - about \( \frac{25}{3} \) hours and 330 miles

3. **You drive to a town 100 miles away. On the trip out you average 50 mi/h. On the trip back you average 50 mi/h. How many hours do you spend driving?**

4. **You drive at an average speed of 50 mi/h to a discount shopping plaza, spend 2 hours shopping, and then return at an average speed of 25 mi/h. The entire trip takes 8 hours. How far away is the shopping plaza?**

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### Graphing Calculator Activity

#### Solution to a System of Linear Equations

A graphing calculator can be used to solve a system of linear equations graphically. The solution of a system of linear equations can be found by using the TRACE feature or by using the Intersect command under the CALC menu.

**Example**

**Solve each system of linear equations.**

1. \( x + y = 0 \)
   - \( x - y = -4 \)

   **Using TRACE:** Solve each equation for \( y \) and enter each equation into \( Y= \). Then graph using Zoom 8: ZInteger. Use TRACE to find the solution.

   **Keystrokes:**
   - \( Y= \)
   - \( 2nd \) \[ \begin{align*} \text{X, \Theta, T, \Pi, } & \quad \text{X=1} \\ \text{CALC} & \quad \text{X=2} \\ \text{TRACE} & \quad \text{X=3} \\ \text{ZOOM} & \quad \text{X=4} \\
   \end{align*} \)

   The solution is \((-2, 2)\).

2. \( 2x + 3y = 4 \)
   - \( 4x + 3y = 3 \)

   **Using CALC:** Solve each equation for \( y \), enter each into the calculator, and graph. Use CALC to determine the solution.

   **Keystrokes:**
   - \( Y= \)
   - \( 2nd \) \[ \begin{align*} \text{X, \Theta, T, \Pi, } & \quad \text{X=1} \\ \text{CALC} & \quad \text{X=2} \\ \text{CALC} & \quad \text{X=3} \\ \text{CALC} & \quad \text{X=4} \\
   \end{align*} \)

   To change the \( x \)-value to a fraction, press \[ \begin{align*} \text{QUIT} & \quad \text{X,T,Ω, \Pi} \\ \text{ZOOM} & \quad \text{Z=1} \\
   \end{align*} \)

   The solution is \((4.5, -5)\) or \((\frac{3}{2}, -\frac{5}{2})\).

### Exercises

**Solve each system of linear equations.**

1. \( y = 2 \)
   - \( 5x + 4y = 18 \)
   - \( (2, 2) \)

2. \( y = -x + 3 \)
   - \( y = x + 1 \)
   - \( (1, 2) \)

3. \( x + y = -1 \)
   - \( 2x - y = -8 \)
   - \( (-3, 2) \)

4. \( 3x + y = 10 \)
   - \( -x + 2y = 0 \)
   - \( (-4, -2) \)

5. \( 5x - 3y = 11 \)
   - \( 7x + y = 20 \)
   - \( (2.5, 0.5) \)

6. \( 6x - 5y = 0 \)
   - \( 3x + 2y = 4 \)
   - \( (5, 4) \)

7. \( -4x + 3y = 5 \)
   - \( -6x - 4y = -8 \)
   - \( infinite solutions \)

8. \( infinite solutions \)

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Glencoe Algebra 1
**Exercise**  
Use substitution to solve each system of equations.  
1. \( y = 4x \)  
   \( 3x - y = 1 \)  
2. \( x = 2y \)  
   \( y = x - 2 \)  
3. \( x = 2y - 3 \)  
4. \( 4x - 2y = 1 \)  
   \( 3x = y + 4 \)  
5. \( x = y - 1 \)  
   \( 3y - x = 2 \)  
6. \( 6x + 3y = 2 \)  
7. \( 2x - y = 4 \)  
   \( x = y - 2 \)  
8. \( x = y - 1 \)  
   \( 3x + 2y = 0 \)  
9. \( 3x + y = 2 \)  
   \( 3y = 5 \)  
10. \( y = 2x \)  
    \( x - 2y = -5 \)  
   \( 0.25x + 0.5y = 10 \)  
   \( x + 2y = -1 \)  
   \( 0.4x + y = 1.1 \)  

---

**Exercises**  
Solve real-world problems.  
1. **SPORTS** At the end of the 2007–2008 football season, 38 Super Bowl games had been played with the current two football leagues, the American Football Conference (AFC) and the National Football Conference (NFC). The AFC won two more games than the NFC. How many games did each conference win?  
2. **CHEMISTRY** A lab needs to make 100 gallons of an 18% acid solution by mixing a 12% acid solution with a 20% solution. How many gallons of each solution are needed?  
3. **GEOMETRY** The perimeter of a triangle is 24 inches. The longest side is 4 inches longer than the shortest side, and the shortest side is three-fourths the length of the middle side. Find the length of each side of the triangle.  

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**Solve by Substitution**  
One method of solving systems of equations is substitution.  

**Example 1** Use substitution to solve the system of equations.  
\[ \begin{align*} 4x - y &= -4 \\
2x - 4y &= -6 \end{align*} \]  
Substitute \( 2x \) for \( y \) in the second equation.  
\[ \begin{align*} 4x - y &= -4 \\
2x &= -4 \end{align*} \]  
The solution is \( (-2, -4) \).  

**Example 2** Solve for one variable, then substitute.  
\[ \begin{align*} x + 3y &= 7 \\
2x - 4y &= -6 \end{align*} \]  
Solve the first equation for \( x \) since the coefficient of \( x \) is 1.  
\[ \begin{align*} x + 3y &= 7 \\
x &= 7 - 3y \end{align*} \]  
Find the value of \( y \) by substituting \( 7 - 3y \) for \( x \) in the second equation.  
\[ \begin{align*} 2x - 4y &= -6 \\
2(7 - 3y) - 4y &= -6 \end{align*} \]  
Simplify.  
\[ \begin{align*} 2x - 4y &= -6 \\
x &= 7 - 3y \end{align*} \]  
\[ \begin{align*} 14 - 10y &= -6 \end{align*} \]  
Divide each side by \(-10\) and simplify.  
\[ \begin{align*} y &= 2 \end{align*} \]  
\[ \begin{align*} x &= 7 - 3(2) \end{align*} \]  
\[ \begin{align*} x &= 7 - 6 \end{align*} \]  
The solution is \( (1, 2) \).
Use substitution to solve each system of equations.

1. \( y = 4x \)
   \[ x + y = 5 \, \text{(1, 4)} \]
   \[ x + 3y = -14 \, (-2, -4) \]

2. \( y = 2x \)
   \[ x + 3y = 20 \, (8, -2) \]

3. \( y = 3x \)
   \[ 2x + y = 15 \, (3, 9) \]
   \[ 3x + 2y = 20 \, (8, -2) \]

4. \( y = -4y \)
   \[ x = 2x - 2 \]
   \[ 3x + 2y = 20 \, (8, -2) \]

5. \( y = x - 1 \)
   \[ x + y = 2 \, (-6, 1) \]

6. \( y = 4x - 1 \)
   \[ 5x + 2y = 5 \, (-1, 5) \]

7. \( y = 5x + 8 \)
   \[ 4x + 3y = 33 \, (3, 7) \]

8. \( y = 2x - 5 \)
   \[ -2x - 3y = -24 \]

9. \( 2x - 3y = 21 \)
   \[ y = 3x - (6, -3) \]

10. \( 5x + 2y = 18 \)
    \[ x = -y - 5 \, (4, 3) \]

11. \( x + 2y = 13 \)
    \[ 3x - 5y = 6 \, (7, 3) \]

12. \( x + 5y = 4 \)
    \[ 3x + 15y = -1 \, \text{no solution} \]

13. \( 3x - y = 4 \)
    \[ 2x - 3y = 10 \, (3, 5) \]

14. \( x + 4y = 8 \)
    \[ 2x - 5y = 29 \, (12, -1) \]

15. \( x - 5y = 10 \)
    \[ 2x - 10y = 20 \, \text{infinitely many} \]

16. \( y = -x - 5 \)
    \[ x + y = 3 \, (2, 3) \]

17. \( 2x + 3y = 38 \)
    \[ x - 3y = -3 \, (9, 4) \]

18. \( x - 4y = 27 \)
    \[ 3x + y = -23 \, (-5, -8) \]

19. \( 2x + 2y = 7 \)
    \[ x - 2y = -1 \left( 2.5, -3.5 \right) \]

20. \( 2.5x + y = -2 \)
    \[ 3x + 2y = 2 \, (-2, 3) \]

---

**Answers**

1. \((1, 4)\)
2. \((9, 3)\)
3. \((1, -3)\)
4. \((2, -1)\)
5. \((4, 6)\)
6. \((7, -9)\)
7. \((-3, 8)\)
8. \((3, 9)\)
9. \((-4, -8)\)
10. \((-6, 4)\)
11. \((14, 5)\)
12. \((5, 5)\)
13. \((-1, 5)\)
14. \((5, 2)\)
15. \((12, 3)\)
16. \((5, 5)\)
17. \((-4, -8)\)
18. \((2, 3)\)
19. \((5, 5)\)
20. \((3, 9)\)

---

**EMPLOYMENT**

Kenisha sells athletic shoes part-time at a department store. She can earn either $500 per month plus a 3% commission on her total sales, or $400 per month plus a 5% commission on total sales.

a. Write a system of equations to represent the situation.
   \[ y = 0.04x + 500 \, \text{and} \, y = 0.05x + 400 \]
b. What is the total price of the athletic shoes Kenisha needs to sell to earn the same income from each pay scale? $10,000

c. Which is the better offer? the first offer if she expects to sell less than $10,000 in shoes, and the second offer if she expects to sell more than $10,000 in shoes

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**MOVIE TICKETS**

Tickets to a movie cost $7.25 for adults and $5.50 for students. A group of friends purchased 8 tickets for $52.75.

a. Write a system of equations to represent the situation.
   \[ x + y = 8 \, \text{and} \, 7.25x + 5.50y = 52.75 \]
b. How many adult tickets and student tickets were purchased?
   5 adult and 3 student tickets

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6-2 Word Problem Practice

1. BUSINESS Mr. Randolph finds that the supply and demand for gasoline at his station are generally given by the following equations.
   \[ x = y - 2 \]
   \[ x + y = 10 \]
   Use substitution to find the equilibrium point where the supply and demand lines intersect. \((4, 6)\)

2. GEOMETRY The measures of complementary angles have a sum of 90 degrees. Angle \(A\) and angle \(B\) are complementary, and their measures have a difference of 20°. What are the measures of the angles? \(35°\) and \(55°\)

3. MONEY Harvey has some $1 bills and some $5 bills. In all, he has 6 bills worth $22. Let \(x\) be the number of $1 bills and let \(y\) be the number of $5 bills. Write a system of equations to represent the information and use substitution to determine how many bills of each denomination Harvey has.
   \[ x + y = 6 \]
   \[ x + 5y = 22 \]
   He has four $5 bills and two $1 bills.

4. POPULATION Sanjay is researching population trends in South America. He found that experts expect the population of Ecuador to increase by 1,000,000 and the population of Chile to increase by 600,000 from 2004 to 2009. The table displays the information he found.

<table>
<thead>
<tr>
<th>Country</th>
<th>2004 Population</th>
<th>Predicted 5-Year Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ecuador</td>
<td>13,000,000</td>
<td>+1,000,000</td>
</tr>
<tr>
<td>Chile</td>
<td>16,000,000</td>
<td>-600,000</td>
</tr>
</tbody>
</table>

   If the population growth for each country continues at the same rate, in what year are the populations of Ecuador and Chile predicted to be equal?

5. CHEMISTRY Shelby and Calvin are doing a chemistry experiment. They need 5 ounces of a solution that is 65% acid and 35% distilled water. There is no pure acid. There is 20% acid and 80% distilled water.

   a. Write a system of equations that Shelby and Calvin could use to determine how many ounces of each flask they need to pour from each flask to make their solution.
   \[
   \begin{align*}
   0.65a + 0.2b &= 5 \\
   0.7a + 0.2b &= 5.5
   \end{align*}
   \]
   Sample answer: \(a + b = 5; 0.7a + 0.2b = 0.55\)

   b. Solve the system of equations. How many ounces from each flask do Shelby and Calvin need? 4.5 oz from Flask A and 0.5 oz from Flask B.

6-2 Enrichment

Intersection of Two Parabolas

Substitution can be used to find the intersection of two parabolas. Replace the \(y\)-value in one of the equations with the \(y\)-value in terms of \(x\) from the other equation.

Example

Find the intersection of the two parabolas.

\[
\begin{align*}
  y &= x^2 + 5x + 6 \\
  y &= x^2 + 4x + 3
\end{align*}
\]

Graph the equations.

From the graph, notice that the two graphs intersect in one point.

Use substitution to solve for the point of intersection.

\[
\begin{align*}
  x^2 + 5x + 6 &= x^2 + 4x + 3 \\
  5x + 6 &= 4x + 3 \\
  x &= -3
\end{align*}
\]

The graphs intersect at \(x = -3\).

Replace \(x\) with \(-3\) in each equation to find the \(y\)-value.

\[
\begin{align*}
  y &= x^2 + 5x + 6 & \text{Original equation} \\
  y &= x^2 + 4x + 3 & \text{Replace } x \text{ with } -3 \\
  y &= (9) + 15 + 6 & \text{Simplify} \\
  y &= 30
\end{align*}
\]

The point of intersection is \((-3, 30)\).

Exercises

Use substitution to find the point of intersection of the graphs of each pair of equations.

\[
\begin{align*}
  1. y &= x^2 + 8x + 7 \\
  2. y &= x^2 + 6x + 8 \\
  3. y &= x^2 + 5x + 6 \\
  y &= x^2 + 2x + 1 & \text{Original equation} \\
  y &= x^2 + 4x + 4 & \text{Replace } x \text{ with } -2 \\
  y &= x^2 + 7x + 6 & \text{Simplify} \\
  (-1, 6) & \text{ or } (0, 6)
\end{align*}
\]

Chapter 6
Elimination Using Addition

In systems of equations where the coefficients of the \(x\) or \(y\) terms are additive inverses, solve the system by adding the equations. Because one of the variables is eliminated, this method is called elimination.

**Example 1**

Use elimination to solve the system of equations.

\[
\begin{align*}
3x + 3y &= 9 \\
2x - 2y &= 2
\end{align*}
\]

Solve for \(x\).

\[
x = \frac{4}{4}
\]

Substitute \(4\) for \(x\) in either equation and solve for \(y\).

\[
y = \frac{3}{3} \quad \text{ or } \quad -\frac{3}{3}
\]

The solution is \((4, -1)\).

**Example 3**

The sum of two numbers is 70 and their difference is 24. Find the numbers.

Let \(x\) represent one number and \(y\) represent the other number.

\[
\begin{align*}
x + y &= 70 \\
(+) x - y &= 24
\end{align*}
\]

\[
\begin{align*}
2x &= 94 \\
x &= 47
\end{align*}
\]

Substitute 47 for \(x\) in either equation.

\[
\begin{align*}
47 + y &= 70 \\
47 + y - 47 &= 70 - 47 \\
y &= 23
\end{align*}
\]

The numbers are 47 and 23.

**Exercises**

Use elimination to solve each system of equations.

1. \(\begin{align*}
x + y &= -4 \\
x - y &= 2
\end{align*}\) \((1, -4)\)
2. \(\begin{align*}
2x - 3y &= 14 \\
x + 3y &= -11
\end{align*}\) \((-2, 3)\)
3. \(\begin{align*}
x - y &= -9 \\
(+) 3x - y &= 9
\end{align*}\) \((-2, 3)\)
4. \(\begin{align*}
x + y &= -4 \\
2x - y &= 6
\end{align*}\) \((2, -2)\)
5. \(\begin{align*}
x &= 2y \\
x + y &= 2
\end{align*}\) \((0, 1)\)
6. \(\begin{align*}
x &= 2y \\
x &= -2y
\end{align*}\) \((-3, 2)\)
7. \(\begin{align*}
x + 2y &= 2 \\
x &= 2
\end{align*}\) \((1, -1)\)
8. \(\begin{align*}
x &= 2 \\
x &= -2
\end{align*}\) \((0, 0)\)
9. \(\begin{align*}
x &= 2y \\
x &= -2
\end{align*}\) \((-2, 2)\)
10. \(\begin{align*}
x &= 3y + 2 \\
x &= -3y + 2
\end{align*}\) \((0, 0)\)

13. Rema is older than Ken. The difference of their ages is 12 and the sum of their ages is 50. Find the age of each. Rema is 31 and Ken is 19.

14. The sum of the digits of a two-digit number is 12. The difference of the digits is 2. Find the number if the units digit is larger than the tens digit. 57
Elimination Using Addition and Subtraction

Use elimination to solve each system of equations.

1. \( x - y = 1 \)
   \[ x + y = 3 \]  \( (2, 1) \)
2. \( x + y = 1 \)
   \[ x + y = 11 \]  \( (6, -5) \)
3. \( x - 4y = 11 \)
   \[ x - 6y = 11 \]  \( (11, 0) \)
4. \( -x + 3y = 6 \)
   \[ x + 3y = 18 \]  \( (6, 4) \)
5. \( 3x + 4y = 19 \)
   \[ 3x + 6y = 33 \]  \( (-3, 7) \)
6. \( x + 4y = 1 \)
   \[ x = 4y = -8 \]  \( (0, -4) \)
7. \( 3x + 4y = 2 \)
   \[ 4x - 4y = 12 \]  \( (2, -1) \)
8. \( 3x - y = -1 \)
   \[ -3x - y = 5 \]  \( (-1, -2) \)
9. \( 2x - 3y = 9 \)
   \[ -5x - 3y = 30 \]  \( (-3, -5) \)
10. \( x - y = 4 \)
    \[ 2x + y = -4 \]  \( (0, -4) \)
11. \( 3x - y = 24 \)
    \[ 2x - y = -24 \]  \( (10, 4) \)
12. \( 5x - y = -6 \)
    \[ -x + y = 2 \]  \( (-1, 1) \)
13. \( 6x - 2y = 32 \)
    \[ 4x - 2y = 18 \]  \( (7, 5) \)
14. \( 3x + 2y = -19 \)
    \[ -3x - 5y = 25 \]  \( (-5, -2) \)
15. \( 7x + 4y = 2 \)
    \[ 7x + 2y = 8 \]  \( (2, -3) \)
16. \( 2x - 5y = -28 \)
    \[ 4x + 9y = 4 \]  \( (-4, 4) \)

17. The sum of two numbers is 28 and their difference is 4. What are the numbers? 12, 16
18. Find the two numbers whose sum is 29 and whose difference is 15. 7, 22
19. The sum of two numbers is 24 and their difference is 2. What are the numbers? 13, 11
20. Find the two numbers whose sum is 54 and whose difference is 4. 25, 29
21. Two times a number added to another number is 25. Three times the first number minus the other number is 20. Find the numbers. 9, 7

Answers

1. \( x = 1 \)
   \[ y = 0 \]
2. \( p = -2 \)
   \[ q = 8 \]
3. \( 4c + y = 23 \)
   \[ 3c - y = 12 \]
4. \( 2x + 5y = -3 \)
   \[ 2x + 2y = -6 \]
5. \( 5x + 3y = 22 \)
   \[ 5x - 2y = 2 \]
6. \( 6x - 3 \)
   \[ 4x + 2y = -6 \]
7. \( 7.5x + 2y = 7 \)
   \[ -2x + 2y = -14 \]
8. \( 3x - 9y = -12 \)
   \[ 2x - 15y = -6 \]
9. \( -4x - 2y = -2 \)
   \[ -3x - 6y = -14 \]
10. \( 10.2x - 6y = 6 \)
    \[ 7x - 2y = -30 \]
11. \( 11.7x + 2y = 2 \)
    \[ 12.4x - 1.29y = -9.2 \]
12. \( 2x + 3y = 24 \)
    \[ 1.6x - 1.25y = 7.6 \]
13. \( 14.5x + 10y = 10 \)
    \[ 15.6x - 8y = 3 \]
14. \( 2.5x + 2y = 12.9 \)
    \[ 20 - 8y = -3 \]
15. \( (2.5, 1.25) \)
    \[ (3.4, 2.2) \]
16. \( 17.1x - 3.3y = -2 \)
    \[ \frac{3}{2}x - \frac{3}{2}y = 8 \]
17. \( 4x + 3y = 10 \)
    \[ \frac{1}{2}x - \frac{1}{2}y = 19 \]
18. \( \left(-\frac{1}{2}, 4\right) \)
    \[ (10, -1) \]
19. The sum of two numbers is 41 and their difference is 5. What are the numbers? 18, 23
20. Four times one number added to another number is 36. Three times the first number minus the other number is 20. Find the numbers. 8, 4
21. One number added to three times another number is 24. Five times the first number added to three times the other number is 36. Find the numbers. 5, 7
22. LANGUAGES
    English is spoken as the first or primary language in 78 more countries than Farsi is spoken as the first language. Together, English and Farsi are spoken as a first language in 130 countries. In how many countries is English spoken as the first language? In how many countries is Farsi spoken as the first language?
    English: 104 countries, Farsi: 26 countries
23. DISCOUNTS
    At a sale on winter clothing, Cody bought two pairs of gloves and four hats for $43.00. Tori bought two pairs of gloves and two hats for $30.00. What were the prices for the gloves and hats?
    gloves: $8.50, hats: $6.50.
Word Problem Practice
Elimination Using Addition and Subtraction

1. NUMBER FUN Ms. Simms, the sixth grade math teacher, gave her students this challenge problem.
   Twice a number added to another number is 15. The sum of the two numbers is 11. Lorenzo, an algebra student who was Ms. Simms aide, realized he could solve the problem by writing the following system of equations.
   \[2x + y = 15\]
   \[x + y = 11\]
   Use the elimination method to solve the system and find the two numbers. \((4, 7)\)

2. GOVERNMENT The Texas State Legislature is comprised of state senators and state representatives. The sum of the number of senators and representatives is 181. There are 119 more representatives than senators. How many senators and how many representatives make up the Texas State Legislature?
   \[S + R = 181\]
   \[R = S + 119\]
   Solving each system of equations. 
   \[S = 22\]
   \[R = 159\]

3. RESEARCH Melissa wondered how much it cost to send a letter by mail in 1990, so she asked her father. Rather than answer directly, Melissa’s father gave her the following information. It would have cost $8.70 to send 13 postcards and 7 letters, and it would have cost $2.05 to send 6 postcards and 7 letters. Use a system of equations and elimination to find how much it cost to send a letter in 1990. \$0.25

   \[13p + 7l = 8.70\]
   \[6p + 7l = 2.05\]

4. SPORTS As of 2007 the New York Yankees had won more Major League Baseball World Series than any other team. In fact, The Yankees had won 1 fewer than 3 times the number of World Series won by the Oakland A’s. The sum of the two teams’ World Series championships is 35. How many times has each team won the World Series?
   \[y + a = 35\]
   \[y = 3(x - 1)\]
   \[2y = 35\]
   \[y = 17\]
   \[a = 18\]

5. BASKETBALL In 2005, the average ticket prices for Dallas Mavericks games and Boston Celtics games are shown in the table below. The change in price is from the 2004 season to the 2005 season.

<table>
<thead>
<tr>
<th>Team</th>
<th>Average Ticket Price</th>
<th>Change in Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dallas</td>
<td>$53.60</td>
<td>$0.53</td>
</tr>
<tr>
<td>Boston</td>
<td>$55.93</td>
<td>$1.08</td>
</tr>
</tbody>
</table>

   \[3x + 2y + z = 42\]
   \[2y + z + 12 = 3x\]
   \[x + 3y = 0\]
   \[x = 9\]
   \[y = 3\]
   \[z = 9\]

   \[x + y + z = -3\]
   \[2x + 3y + 5z = -4\]
   \[x + 2y + z = 7\]
   \[2y + z = 5\]
   \[x = 5\]
   \[y = 3\]
   \[z = -1\]

   \[3x + y + z = 7\]
   \[2y + z = 4\]
   \[2y + z = 5\]
   \[x = 5\]
   \[y = 3\]
   \[z = -1\]
**Elimination Using Multiplication**

Some systems of equations cannot be solved simply by adding or subtracting the equations. In such cases, one or both equations must first be multiplied by a number before the system can be solved by elimination.

**Example 1**

Use elimination to solve the system of equations.

\[\begin{align*}
4x + 5y &= 5 \\
6x - 7y &= -20
\end{align*}\]

If you multiply the second equation by \(-2\), you can eliminate the \(y\) terms.

\[\begin{align*}
x + 10y &= 3 \\
(-2) \cdot (6x - 7y) &= -20 \cdot (-2)
\end{align*}\]

Substitute 1 for \(x\) in either equation.

\[\begin{align*}
1 + 10y &= 3 \\
10y &= 2 \\
y &= \frac{1}{5}
\end{align*}\]

The solution is \((\frac{1}{5}, 1)\).

**Example 2**

Use elimination to solve the system of equations.

\[\begin{align*}
2x - 3y &= -4 \\
3x + 2y &= 3
\end{align*}\]

Substitute \(-4\) for \(y\) in either equation.

\[\begin{align*}
3x + 8 - 8 &= -7 - 8 \\
x + 3y &= -10
\end{align*}\]

The solution is \((-5, -4)\).

**Exercises**

Use elimination to solve each system of equations.

1. \(2x + 3y = 6 \quad x + 2y = 5\)
2. \(2m + 3n = 4 \quad -m + 2n = 5\)
3. \(3a - b = 2 \quad a + 2b = 3\)
4. \(4x + 5y = 6 \quad 6x - 7y = -20\)
5. \(4a - 3b = -8 \quad 2a + 2b = 3\)
6. \(2x - y = 9 \quad 5x + 2y = 8\)
7. \(4x + 2y = -3 \quad 11x + 4y = -5\)
8. \(|x| \leq 2\)

13. **GARDENING** The length of Sally’s garden is 4 meters greater than 3 times the width. The perimeter of her garden is 72 meters. What are the dimensions of Sally’s garden? 28 m by 8 m

**Answers**

- Example 3:
  \[\begin{align*}
x &= \frac{10}{3} \\
y &= \frac{1}{3}
\end{align*}\]

- Example 4:
  \[\begin{align*}
x &= \frac{5}{3} \\
y &= \frac{2}{3}
\end{align*}\]

- Example 5:
  \[\begin{align*}
x &= \frac{3}{2} \\
y &= \frac{1}{2}
\end{align*}\]

- Example 6:
  \[\begin{align*}
x &= \frac{1}{2} \\
y &= \frac{1}{2}
\end{align*}\]

- Example 7:
  \[\begin{align*}
x &= \frac{1}{3} \\
y &= \frac{1}{3}
\end{align*}\]

- Example 8:
  \[\begin{align*}
x &= \frac{1}{2} \\
y &= \frac{1}{2}
\end{align*}\]

- Example 9:
  \[\begin{align*}
x &= \frac{1}{2} \\
y &= \frac{1}{2}
\end{align*}\]

- Example 10:
  \[\begin{align*}
x &= \frac{1}{2} \\
y &= \frac{1}{2}
\end{align*}\]

- Example 11:
  \[\begin{align*}
x &= \frac{1}{2} \\
y &= \frac{1}{2}
\end{align*}\]

- Example 12:
  \[\begin{align*}
x &= \frac{1}{2} \\
y &= \frac{1}{2}
\end{align*}\]
Elimination Using Multiplication

Use elimination to solve each system of equations.

1. \(x + y = -9\)
2. \(3x + 2y = -9\)
   \(5x - 2y = 32\)  \((2, -11)\)
3. \(2x + 5y = 3\)
   \(-x + 3y = -7\)  \((4, -1)\)
4. \(2x + y = 3\)
   \(-4x - 4y = -8\)  \((1, 1)\)
5. \(4x - 2y = -14\)
   \(3x - y = -8\)  \((-1, 5)\)
6. \(2x + y = 0\)
   \(5x + 3y = 2\)  \((-2, 4)\)
7. \(5x + 3y = -10\)
   \(2x + 3y = 14\)  \((-2, 0)\)
8. \(2x + y = 1\)
   \(4x - 4y = 9\)  \((4, 2)\)
9. \(2x - 3y = 21\)
   \(5x - 2y = 25\)  \((3, -5)\)
10. \(3x + 2y = -26\)
    \(4x - 4y = -4\)  \((-6, -4)\)
11. \(3x - 6y = -3\)
    \(2x + 4y = 30\)  \((7, 4)\)
12. \(5x + 2y = -3\)
    \(3x + 3y = 9\)  \((-3, 6)\)

13. Eight times a number plus five times another number equals 13. The sum of the two numbers is 7. What are the numbers? 8, -1
14. Four times a number minus twice another number is -16. The sum of the two numbers is -1. Find the numbers. -3, 2
15. Trisha and Byron are washing and vacuuming cars to raise money for a class trip. Trisha raised $38 washing 5 cars and vacuuming 4 cars. Byron raised $28 by washing 4 cars and vacuuming 2 cars. Find the amount they charged to wash a car and vacuum a car.
   wash: $6, vacuum: $2

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6-4 Word Problem Practice

Elimination Using Multiplication

1. SOCCER Suppose a youth soccer field has a perimeter of 320 yards and its length measures 40 yards more than its width. Ms. Hughley asks her players to determine the length and width of their field. She gives them the following system of equations to represent the situation. Use elimination to solve the system to find the length and width of the field.

\[
\begin{align*}
2L + 2W &= 320 \\
L - W &= 40
\end{align*}
\]

**Answer:**

width = 60 yd; length = 100 yd

2. SPORTS The Fan Cost Index (FCI) tracks the average costs for attending sporting events, including tickets, drinks, food, parking, programs, and souvenirs. According to the FCI, a family of four would spend a total of $912.30 to attend two Major League Baseball (MLB) games and one National Basketball Association (NBA) game. The family would spend $691.31 to attend one MLB and two NBA games. Write and solve a system of equations to find the family’s costs for each kind of game according to the FCI.

\[
\begin{align*}
x - y &= 10 \\
x + 3y &= 8
\end{align*}
\]

**Answer:**

NBA: $283.44; MLB: $164.43

3. ART Mr. Santos, the curator of the children’s museum, recently made two purchases of clay and wood for a visiting artist to sculpt. Use the table to find the cost of each product per kilogram.

<table>
<thead>
<tr>
<th>Clay (kg)</th>
<th>Wood (kg)</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
<td>$35.50</td>
</tr>
<tr>
<td>3.5</td>
<td>6</td>
<td>$50.45</td>
</tr>
</tbody>
</table>

Clay was $0.70 per kilogram and wood was $8.00 per kilogram.

4. TRAVEL Antonio flies from Houston to Philadelphia, a distance of about 1340 miles. His plane travels with the wind and takes 2 hours and 20 minutes. At the same time, Paul’s flight takes 2 hours and 50 minutes. What was the speed of the wind in miles per hour?

**Answer:**

about 50.67 mph

5. BUSINESS Suppose you start a business assembling and selling motorized scooters. It costs you $1500 for tools and equipment to get started, and the materials cost $200 for each scooter. Your scooters sell for $300 each.

\[
\begin{align*}
x - y &= 10 \\
y &= 300x + 200
\end{align*}
\]

**Answer:**

Sample answer: 20 scooters: It costs $5500 to assemble these scooters, which sell for $6000, leaving a $500 profit.

6-4 Enrichment

George Washington Carver and Percy Julian

In 1990, George Washington Carver and Percy Julian became the first African Americans elected to the National Inventors Hall of Fame. Carver (1864–1943) was an agricultural scientist known worldwide for developing hundreds of uses for the peanut and the sweet potato. His work revitalized the economy of the southern United States because it was no longer dependent solely upon cotton. Julian (1898–1975) was a research chemist who became famous for inventing a method of making a synthetic cortisone from soybeans. His discovery has had many medical applications, particularly in the treatment of arthritis.

There are dozens of other African American inventors whose accomplishments are not as well known. Their inventions range from common household items like the ironing board to complex devices that have revolutionized manufacturing. The exercises that follow will help you identify just a few of these inventors and their inventions.

Match the inventors with their inventions by matching each system with its solution. (Not all the solutions will be used.)

<table>
<thead>
<tr>
<th>Inventor</th>
<th>Equation 1</th>
<th>Equation 2</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. (1, 4)</td>
<td>automatic traffic signal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B. (4, -2)</td>
<td>eggbeater</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C. (-2, 3)</td>
<td>fire extinguisher</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D. (-5, 7)</td>
<td>folding cabinet bed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E. (6, -4)</td>
<td>ironing board</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F. (-2, 4)</td>
<td>pencil sharpener</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G. (-3, 0)</td>
<td>portable x-ray machine</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H. (2, -3)</td>
<td>player piano</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I. no solution</td>
<td>evaporating pan for refining sugar</td>
<td></td>
<td></td>
</tr>
<tr>
<td>J. infinitely many solutions</td>
<td>lasting (shaping) machine for manufacturing shoes</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Applying Systems of Linear Equations

Determine The Best Method

You have learned five methods for solving systems of linear equations: graphing, substitution, elimination using addition, elimination using subtraction, and elimination using multiplication. For an exact solution, an algebraic method is best.

Example

At a baseball game, Henry bought 3 hotdogs and a bag of chips for $14. Scott bought 2 hotdogs and a bag of chips for $10. Each of the boys paid the same price for their hotdogs, and the same price for their chips. The following system of equations can be used to represent the situation. Determine the best method to solve the system of equations. Then solve the system.

\[ 3x + y = 14 \]
\[ 2x + y = 10 \]

- Since neither the coefficients of \( x \) nor the coefficients of \( y \) are additive inverses, you cannot use elimination using addition.
- Since the coefficient of \( y \) in both equations is 1, you can use elimination using subtraction. You could also use the substitution method or elimination using multiplication.

The following solution uses elimination by subtraction to solve this system.

\[ \begin{align*}
3x + y &= 14 \\
2x + y &= 10 \\
\Rightarrow (3x + y) - (2x + y) &= 14 - 10 \\
x &= 4 \\
3(4) + y &= 14 \\
y &= 2
\end{align*} \]

This means that hot dogs cost $4 each and a bag of chips costs $2.

Exercises

Determine the best method to solve each system of equations. Then solve the system.

1. \[ 5x + 3y = 16 \\
   3x - 5y = -4 \]
   elimination \((x)\); \((2, 2)\)

2. \[ 2x - 5y = 7 \\
   2x + 5y = 13 \]
   elimination \((+)\); \((4, 1)\)

3. \[ y + 3x = 24 \\
   5x - y = 8 \]
   elimination \((+)\); \((4, 12)\)

4. \[ -1x - 10y = 17 \\
   5x - 7y = 50 \]
   elimination \((x)\); \((3, -5)\)

Apply Systems Of Linear Equations

When applying systems of linear equations to problem situations, it is important to analyze each solution in the context of the situation.

- Since neither the coefficients of \( x \) nor the coefficients of \( y \) are additive inverses, you cannot use elimination using addition.
- Since the coefficient of \( y \) in both equations is 1, you can use elimination using subtraction. You could also use the substitution method or elimination using multiplication.

Example

T-shirt Printing Cost

A T-shirt printing company sells T-shirts for $8 each. The company has a fixed cost of $5000 for the printing machine and $10 per T-shirt. The company must sell 1000 T-shirts in order to have an income equal to expenses.

Understand

You know the initial income and the initial expense and the rates of change of each quantity with each T-shirt sold.

Plan

Write an equation to represent the income and the expenses. Then solve to find how many T-shirts need to be sold for both values to be equal.

Solve

Let \( x \) be the number of T-shirts sold and let \( y \) be the total amount.

\[
\begin{align*}
\text{income: } y &= 0 + 8x \\
\text{expenses: } y &= 5000 + 10x
\end{align*}
\]

You can use substitution to solve this system.

\[ y = 15x \]

Substitute the value for \( y \) into the second equation.

\[ 15x = 3000 + 5x \]

Subtract 10\(x\) from each side and simplify.

\[ 10x = 3000 \]

Divide each side by 10 and simplify.

\[ x = 300 \]

This means that if 300 T-shirts are sold, the income and expenses of the T-shirt company are equal.

Check

Does this solution make sense in the context of the problem? After selling 100 T-shirts, the income would be about 100 \(\times\) \$8 or \$800. The costs would be about \$5000 + 100 \(\times\) \$10 or \$1000.

Exercises

Refer to the example above. If the costs of the T-shirt company change to the given values and the selling price remains the same, determine the number of T-shirts the company must sell in order for income to equal expenses.

1. printing machine: $5000.00; T-shirt: $10.00 each
   2. printing machine: $2100.00; T-shirt: $8.00 each
   3. printing machine: $8400.00; T-shirt: $4.00 each
   4. printing machine: $1200.00; T-shirt: $12.00 each
6-5 Skills Practice
Applying Systems of Linear Equations

Determine the best method to solve each system of equations. Then solve the system.

1. \(5x + 3y = 16\)
   \(3x - 5y = -4\)
   elimination (\(x\)); (2, 2)

2. \(3x - 5y = 7\)
   \(2x + 5y = 13\)
   elimination (\(+\)); (4, 1)

3. \(y = 3x - 24\)
   \(5x - y = 8\)
   substitution; \((-8, -48)\)

4. \(-11x - 10y = 17\)
   \(5x - 7y = 50\)
   elimination (\(x\)); (3, -5)

5. \(4x + y = 24\)
   \(5x - y = 12\)
   elimination (\(+\)); (4, 8)

6. \(6x - y = -145\)
   \(x = 4 - 2y\)
   substitution; \((-22, 13)\)

7. VEGETABLE STAND A roadside vegetable stand sells pumpkins for $5 each and squashes for $3 each. One day they sold 6 more squash than pumpkins, and their sales totaled $98. Write and solve a system of equations to find how many pumpkins and squashes they sold.
   \[y = 6 + x \text{ and } 5x + 3y = 98; 10 \text{ pumpkins, 16 squashes}\]

8. INCOME Ramiro earns $20 per hour during the week and $30 per hour for overtime on the weekends. One week Ramiro earned a total of $650. He worked 5 times as many hours during the week as he did on the weekend. Write and solve a system of equations to determine how many hours of overtime Ramiro worked on the weekend.
   \[20x + 30y = 650 \text{ and } x = 5y; 5 \text{ hours}\]

9. BASKETBALL Anya makes 14 baskets during her game. Some of these baskets were worth 2-points and others were worth 3-points. In total, she scored 30 points. Write and solve a system of equations to find how 2-points baskets she made.
   \[x + y = 14 \text{ and } 2x + 3y = 30; 12\]

6-5 Practice
Applying Systems of Linear Equations

Determine the best method to solve each system of equations. Then solve the system.

1. \(1.5x - 1.9y = -29\)
   \(x - 0.9y = 4.5\)
   substitution;
   \((63, 65)\)

2. \(2.1x - 0.8y = -312\)
   \(4.8x + 2.4y = 60\)
   elimination (\(x\));
   \((5, 15)\)

3. \(3.6x + 0.7y = 14\)
   \(5.3x - 4y = 43.5\)
   substitution;
   \((12, 7)\)

4. \(14x + 7y = 217\)
   \(14x + 3y = 189\)
   elimination (\(-\));
   \((18.7, 5)\)

5. \(5x - y = 8\)
   \(5x + 0.2y = 38.4\)
   substitution;
   \((15, 9)\)

7. BOOKS A library contains 2000 books. There are 3 times as many non-fiction books as fiction books. Write and solve a system of equations to determine the number of non-fiction and fiction books.
   \[x + y = 2000 \text{ and } x = 3y; 1500 \text{ non-fiction, } 500 \text{ fiction}\]

8. SCHOOL CLUBS The chess club has 16 members and gains a new member every month. The film club has 4 members and gains 4 new members every month. Write and solve a system of equations to find when the number of members in both clubs will be equal.
   \[y = 16 + x \text{ and } y = 4 + 4x; 4 \text{ months}\]

9. Tia and Ken each sold snack bars and magazine subscriptions for a school fund-raiser, as shown in the table. Tia earned $132 and Ken earned $190.

<table>
<thead>
<tr>
<th>Item</th>
<th>Tia Number Sold</th>
<th>Ken Number Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>snack bars</td>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>magazine subscriptions</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

   a. Define variable and formulate a system of linear equation from this situation.
   Let \(x = \text{ the cost per snack bar and let } y = \text{ the cost per magazine subscription; } 16x + 4y = 132\)
   and \(20x + 6y = 190\).

   b. What was the price per snack bar? Determine the reasonableness of your solution. \$2
### 6-5 Word Problem Practice

**Applying Systems of Linear Equations**

1. **MONEY** Veronica has been saving dimes and quarters. She has 94 coins in all, and the total value is $19.30. How many dimes and how many quarters does she have? 28 dimes; 66 quarters.

2. **CHEMISTRY** How many liters of 15% acid and 33% acid should be mixed to make 40 liters of 21% acid solution?

3. **BUILDINGS** The Sears Tower in Chicago is the tallest building in North America. The total height of the tower \( t \) and the antenna that stands on top of it \( a \) is 1729 feet. The difference in heights between the building and the antenna is 1171 feet. How tall is the Sears Tower?

4. **PRODUCE** Roger and Trevor went shopping for produce on the same day. They each bought some apples and some potatoes. The amount they bought and the total price they paid are listed in the table below. What was the price of apples and potatoes per pound? Apples: $1.49 per lb; Potatoes: $0.99 per lb.

- | Apples (lb) | Potatoes (lb) | Total Cost ($) |
- |----|----|-------------|
- | Roger | 8 | 7 | 18.85 |
- | Trevor | 2 | 10 | 12.88 |

5. **SHOPPING** Two stores are having a sale on T-shirts that normally sell for $20. Store S is advertising a \( t \) dollar discount, and Store T is advertising a \( t \) percent discount. Rose spends $63 for three T-shirts from Store S and four from Store T. Find the discount at each store. Store S: 20%; Store T: $5.

6. **TRANSPORTATION** A Speedy River barge bound for New Orleans leaves Baton Rouge, Louisiana, at 9:00 a.m. and travels at a speed of 10 miles per hour. A Rail Transport freight train also bound for New Orleans leaves Baton Rouge at 1:30 p.m. the same day. The train travels at 25 miles per hour, and the river barge travels at 10 miles per hour. Both the barge and the train will travel 100 miles to reach New Orleans.

   a. How far will the train travel before catching up to the barge? 75 mi.

   b. Which shipment will reach New Orleans first? At what time? The train will arrive first. It will arrive in New Orleans at 5:30 P.M. of the same day.

   c. If both shipments take an hour to unload before heading back to Baton Rouge, what is the earliest time that either one of the companies can begin to load grain to ship in Baton Rouge? 1450 ft.

### 6-5 Enrichment

**Cramer’s Rule**

Cramer’s Rule is a method for solving a system of equations. To use Cramer’s Rule, set up a matrix to represent the equations. A matrix is a way of organizing data.

**Example**

Solve the following system of equations using Cramer’s Rule.

\[ \begin{align*}
2x + 3y &= 13 \\
2x + y &= 5
\end{align*} \]

**Steps**

1. Set up a matrix representing the coefficients of \( x \) and \( y \).

\[ A = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix} \]

2. Find the determinant of matrix \( A \).

\[ \text{det}(A) = 2(1) - 1(3) = -1 \]

3. Find the determinant of the matrix \( A \) with 13 and 5.

\[ \text{det}(A) = 2(13) - 1(5) = 11 \]

4. To find the value of \( x \) in the solution to the system of equations, determine the value of \( \text{det}(A) \).

\[ \text{det}(A) = 11 \]

5. Repeat the process to find the value of \( y \). This time, replace the second column with 13 and 5 and find the determinant.

\[ \text{det}(A) = 2(13) - 1(5) = 21 \]

So, the solution to the system of equations is \((2, 3)\).
Organizing Data Using Matrices

Example:
State the dimensions of the matrix. Then identify the position of the circled element in the matrix.

\[
A = \begin{bmatrix}
-1 & 0 \\
2 & 2 & 7
\end{bmatrix}
\]

Matrix \(A\) has 2 rows and 3 columns. Therefore, it is a \(2 \times 3\) matrix. The circled element is in the first row and third column.

Exercises:
State the dimensions of each matrix. Then identify the position of the circled element in each matrix.

1. \(\begin{bmatrix} 9 & 1 - 2 \ 0 & 14 \end{bmatrix}\)
   - 2 \(\times 4\); second row, second column
   - 2 \(\times 3\); third row, second column
   - 1 \(\times 5\); first row, second column

2. \(\begin{bmatrix} 2 & 10 & -3 \\
-1 & 1 & 5 \\
-4 & 0 & 8
\end{bmatrix}\)
   - 3 \(\times 3\)

4. The selling prices for various luxury condominiums are listed in the table to the right.
   - a. Write a matrix to organize the selling prices of the condos.
   - b. What are the dimensions of the matrix?
   - c. Which condo is the most expensive? least expensive?

<table>
<thead>
<tr>
<th>Condo Development</th>
<th>1 Bedroom</th>
<th>2 Bedrooms</th>
<th>3 Bedrooms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foxpointe Estates</td>
<td>$349,000</td>
<td>$449,000</td>
<td>$499,000</td>
</tr>
<tr>
<td>Condos at Salmon Brook</td>
<td>$329,900</td>
<td>$389,900</td>
<td>$439,900</td>
</tr>
<tr>
<td>Kean Mills</td>
<td>$499,000</td>
<td>$649,000</td>
<td>$799,000</td>
</tr>
</tbody>
</table>

Matrix Operations
If two matrices have the same dimensions, they can be added or subtracted. You add or subtract matrices by adding or subtracting the corresponding elements of the matrices. You can also perform a scalar multiplication by multiplying each element of the matrix by a constant.

Example 1:
If \(A = \begin{bmatrix} 2 & -1 & 8 \\
-3 & 4 & 6
\end{bmatrix}\) and \(B = \begin{bmatrix} 0 & 2 & -4 \\
5 & 6 & 3
\end{bmatrix}\), find \(A + B\).

\[
A + B = \begin{bmatrix} 2 & -1 & 8 \\
-3 & 4 & 6
\end{bmatrix} + \begin{bmatrix} 0 & 2 & -4 \\
5 & 6 & 3
\end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\
2 & -2 & 3
\end{bmatrix}
\]

Example 2:
If \(C = \begin{bmatrix} 2 & -10 \\
-3 & 1
\end{bmatrix}\), find \(-3C\).

\[
-3C = \begin{bmatrix} -6 & 30 \\
9 & -3
\end{bmatrix}
\]

Exercises:
Perform the indicated matrix operations. If the matrix does not exist, write impossible.

1. \(\begin{bmatrix} 3 & -5 & 7 \\
2 & 0 & 3
\end{bmatrix} - \begin{bmatrix} -1 & 4 & 2 \\
9 & 3 & 1
\end{bmatrix} \) impossible

3. \(\begin{bmatrix} 2 & -1 \\
5 & 0
\end{bmatrix} + \begin{bmatrix} 1 & 2 \\
-3 & 4
\end{bmatrix} \)

4. \(\begin{bmatrix} 9 & -12 \\
0 & 4
\end{bmatrix} - \begin{bmatrix} 3 & 2 \\
-4 & 3
\end{bmatrix} \)

5. \(\begin{bmatrix} 2 & 4 \\
0 & -1
\end{bmatrix} - \begin{bmatrix} 4 & 8 \\
0 & -2
\end{bmatrix} \)

6. \(\begin{bmatrix} 0 & 1 \\
2 & 5
\end{bmatrix} - \begin{bmatrix} 0 & 3 \\
6 & 15
\end{bmatrix} \)

7. \(\begin{bmatrix} -2 & 1 \\
-2 & 4
\end{bmatrix} - \begin{bmatrix} 4 & 0 \\
-8 & 0
\end{bmatrix} \)

8. \(\begin{bmatrix} -3 & 2 & -1 \\
1 & 0 & 4
\end{bmatrix} - \begin{bmatrix} -15 & 10 & -5 \\
5 & 0 & 20
\end{bmatrix} \)
### 6-6 Practice

#### Organizing Data Using Matrices

**Exercises**

State the dimensions of each matrix. Then identify the position of the circled element in each matrix.

1. \[ \begin{bmatrix} 1 -1 & 3 \end{bmatrix} \]

   - First row, second column

2. \[ \begin{bmatrix} 2 -0 & -1 & -4 \end{bmatrix} \]

   - Third row, first column

3. \[ \begin{bmatrix} 6 -3 & 0 \end{bmatrix} \]

   - Second row, first column

4. \[ \begin{bmatrix} 1 -2 & -5 \end{bmatrix} \]

   - Third row, second column

5. \[ \begin{bmatrix} 1 -4 & 2 \end{bmatrix} \]

   - Second row, first column

6. \[ \begin{bmatrix} 1 -1 & 0 \end{bmatrix} \]

   - First row, second column

7. \[ \begin{bmatrix} 1 -3 & 2 \end{bmatrix} \]

   - First row, second column

8. \[ \begin{bmatrix} 1 -4 & 3 \end{bmatrix} \]

   - Second row, first column

9. \[ \begin{bmatrix} 1 -1 & 0 \end{bmatrix} \]

   - First row, second column

10. \[ \begin{bmatrix} 1 -1 & 0 \end{bmatrix} \]

    - First row, second column

11. \[ \begin{bmatrix} 1 -1 & 0 \end{bmatrix} \]

    - First row, second column

12. \[ \begin{bmatrix} 1 -1 & 0 \end{bmatrix} \]

    - First row, second column

13. \[ \begin{bmatrix} 1 -1 & 0 \end{bmatrix} \]

    - First row, second column

#### Answers (Lesson 6-6)

**Exercises**

State the dimensions of each matrix. Then identify the position of the circled element in each matrix.

1. \[ \begin{bmatrix} 0 & 1 -4 & 9 \end{bmatrix} \]

   - Second row, first column

2. \[ \begin{bmatrix} 0 & 0 & -3 \end{bmatrix} \]

   - Third row, second column

3. \[ \begin{bmatrix} 9 & 0 & 0 \end{bmatrix} \]

   - First row, second column

4. \[ \begin{bmatrix} 0 & -4 \end{bmatrix} \]

   - Second row, first column

5. \[ \begin{bmatrix} 0 & 3 \end{bmatrix} \]

   - First row, second column

6. \[ \begin{bmatrix} 0 & 3 \end{bmatrix} \]

   - First row, second column

7. \[ \begin{bmatrix} 0 & 3 \end{bmatrix} \]

   - First row, second column

8. \[ \begin{bmatrix} 0 & 3 \end{bmatrix} \]

   - First row, second column

9. \[ \begin{bmatrix} 0 & 3 \end{bmatrix} \]

   - First row, second column

10. \[ \begin{bmatrix} 0 & 3 \end{bmatrix} \]

    - First row, second column

11. \[ \begin{bmatrix} 0 & 3 \end{bmatrix} \]

    - First row, second column

12. \[ \begin{bmatrix} 0 & 3 \end{bmatrix} \]

    - First row, second column

13. \[ \begin{bmatrix} 0 & 3 \end{bmatrix} \]

    - First row, second column

---

**FOOD SALES**

The daily sales at various fast food restaurants in various cities are shown in the table below.

<table>
<thead>
<tr>
<th>City</th>
<th>McPizza</th>
<th>Burger Hut</th>
<th>QuizSubs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Las Vegas</td>
<td>25,000</td>
<td>17,400</td>
<td>21,000</td>
</tr>
<tr>
<td>Phoenix</td>
<td>3,600</td>
<td>4,400</td>
<td>5,900</td>
</tr>
<tr>
<td>Newton</td>
<td>19,200</td>
<td>20,100</td>
<td>17,400</td>
</tr>
</tbody>
</table>

a. Write a matrix to organize the sales data.

b. What is the dimension of the matrix? 3 x 3

c. In which city does Burger Hut sell more food than its competitors? Newton
**6-6 Word Problem Practice**

**Organizing Data Using Matrices**

1. **AP EXAMS** The College Board regularly administers Advanced Placement (AP) exams to students in high schools nationwide. The highest score possible on an AP exam is a 5. Below is a table showing the percentage of students taking selected AP exams in 2007 and the percentage of students who achieved various scores.

<table>
<thead>
<tr>
<th>AP Grade</th>
<th>Calculus AB</th>
<th>Calculus BC</th>
<th>Chemistry</th>
<th>Chinese</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 points</td>
<td>21.0%</td>
<td>43.5%</td>
<td>15.3%</td>
<td>81.0%</td>
</tr>
<tr>
<td>4 points</td>
<td>18.7%</td>
<td>17.9%</td>
<td>18.0%</td>
<td>11.8%</td>
</tr>
</tbody>
</table>

Source: The College Board

Write a matrix to organize the test scores, and state its dimensions. In which test did most students score a 5?

2. **STOCK PRICES** The chart below shows the performance of one share of PQR Corp. and ABC Corp. over the last week.

Organize the data in the chart into a matrix and perform the matrix operation to find the value of 50 shares of PQR Corp. and ABC Corp. on each day.

3. **BASEBALL** In Tuesday's baseball game, Reggie scored 1 run and had 4 hits and 1 stolen base. Jeremy scored 2 runs, had 5 hits, and stole no bases. In Wednesday's baseball game, Reggie didn't score any runs, but did have 3 hits and 1 stolen base. Jeremy had 1 run, 4 hits, and 1 stolen base.

a. Create a matrix showing Reggie and Jeremy's performance in Tuesday's baseball game.

\[
\begin{bmatrix}
1 & 4 & 1 \\
2 & 5 & 0
\end{bmatrix}
\]

b. Create a matrix showing Reggie and Jeremy's performance in Wednesday's baseball game.

\[
\begin{bmatrix}
0 & 3 & 1 \\
1 & 4 & 1
\end{bmatrix}
\]

c. Add the two matrices together to show Reggie and Jeremy's total performance over both games.

\[
\begin{bmatrix}
1 & 7 & 2 \\
3 & 9 & 1
\end{bmatrix}
\]

d. Subtract the matrix representing Tuesday's game from the matrix representing Wednesday's game to show Reggie and Jeremy's change in performance.

\[
\begin{bmatrix}
-1 & 1 & 0 \\
-1 & 1 & 1
\end{bmatrix}
\]

**6-6 Enrichment**

**Multiplying 2 \times 2 Matrices**

Multiplication with matrices is not limited to scalars. A matrix can also be multiplied by another matrix. To find the product of two \(2 \times 2\) matrices, the following equation can be applied:

\[
\begin{bmatrix}
q & r \\
4 & 5
\end{bmatrix}
\begin{bmatrix}
x & y \\
6 & 7
\end{bmatrix}
= \begin{bmatrix}
(qx + ry) & (qx + rz) \\
(6x + 7y) & (6y + 7z)
\end{bmatrix}
\]

**Example 1**

Find \(AB\).

\[
A = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix}
\]

**Example 2**

Find \(BA\).

\[
A = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix}
\]

**Exercises**

1. \[
\begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}
\]

2. \[
\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}
\]

3. \[
\begin{bmatrix} 4 & 1 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}
\]

4. \[
\begin{bmatrix} -1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}
\]

5. \[
\begin{bmatrix} 4 & 1 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}
\]

6. \[
\begin{bmatrix} -1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}
\]

7. If matrix \(C = \begin{bmatrix} c & d \\ f & g \end{bmatrix}\) and matrix \(D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\) then multiply by matrix \(D\) will return a value of \(CD\) equal to the initial matrix \(C\).
Using Matrices to Solve Systems of Equations

**Example 1**
Write an augmented matrix for each system of equations.

1. \(2x + 3y = 8\)
   \(-x + 4y = 7\)

Place the coefficients of the equations and the constant terms into a matrix.

\[
\begin{bmatrix}
2 & 3 & | & 8 \\
-1 & 4 & | & 7
\end{bmatrix}
\]

**Example 2**
Write an augmented matrix for each system of equations.

2. \(x + 3y = 6\)
   \(2y = 6\)

Place the coefficients of the equations and the constant terms into a matrix.

\[
\begin{bmatrix}
1 & 3 & | & 6 \\
0 & 2 & | & 6
\end{bmatrix}
\]

**Exercises**

1. \(2x - 2y = 10\)
2. \(x - y = 3\)
3. \(-x = -4\)

\[
\begin{bmatrix}
2 & -2 & | & 10 \\
-1 & 1 & | & 3 \\
1 & -2 & | & 1
\end{bmatrix}
\]

4. \(-x + y = 9\)
5. \(-2x + 5y = 11\)
6. \(2y = 8\)

\[
\begin{bmatrix}
-1 & 1 & | & 9 \\
-2 & 5 & | & 11 \\
3 & -1 & | & 0 \\
4 & -1 & | & -6
\end{bmatrix}
\]

7. \(x - y = 3\)
8. \(2x - y = 1\)
9. \(2x - y = 4\)

\[
\begin{bmatrix}
1 & -1 & | & 3 \\
2 & 1 & | & 4 \\
0 & 12 & | & 3
\end{bmatrix}
\]

10. \(-2x = -4\)
11. \(5x - 2y = 7\)
12. \(x - y = 5\)

\[
\begin{bmatrix}
-2 & 0 & | & -4 \\
5 & -2 & | & 7 \\
1 & 3 & | & 8 \\
10 & -1 & | & 24
\end{bmatrix}
\]

13. \(x + 4y = 10\)
14. \(3x - y = 1\)
15. \(4y = -2\)

\[
\begin{bmatrix}
1 & 4 & | & 10 \\
[3 & -1 & | & 1] \\
1 & -1 & | & 0 \\
1 & 2 & | & 19
\end{bmatrix}
\]

**Example**
Use an augmented matrix to solve the system of equations.

\[
\begin{bmatrix}
2 & 1 & | & 5 \\
1 & 3 & | & 6
\end{bmatrix}
\]

Notice that the first element in the second row is 1. Interchange the rows so 1 can be in the upper left-hand corner.

\[
\begin{bmatrix}
2 & 1 & | & 5 \\
1 & 3 & | & 6
\end{bmatrix}
\]

To make the first element in the second row 0, multiply the first row by \(-2\) and add the result to row 2.

\[
\begin{bmatrix}
2 & 1 & | & 5 \\
1 & 3 & | & 6
\end{bmatrix}
\]

To make the second element in the second row a 1, multiply the second row by \(-\frac{1}{2}\).

\[
\begin{bmatrix}
2 & 1 & | & 5 \\
1 & 3 & | & 6
\end{bmatrix}
\]

To make the second element in the first row a 0, multiply the second row by \(-\frac{1}{2}\) and add the result to row 1.

\[
\begin{bmatrix}
2 & 1 & | & 5 \\
1 & 3 & | & 6
\end{bmatrix}
\]

The solution is (3, 1).

**Exercises**

Use an augmented matrix to solve each system of equations.

1. \(x + 5y = -7\)
2. \(2x - 2y = 6\)
3. \(-x = -3\)

\[
\begin{bmatrix}
1 & 5 & | & -7 \\
2 & -2 & | & 6 \\
3 & 6 & | & 0
\end{bmatrix}
\]

4. \(-x + y = 9\)
5. \(-2x + 2y = -8\)
6. \(2x = 10\)

\[
\begin{bmatrix}
-1 & 1 & | & 9 \\
-2 & 2 & | & -8 \\
4 & -1 & | & 2
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 10 & | & (1) \\
0 & -4 & | & (0) \\
-2 & 5 & | & (-2)
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 3 & | & 6 \\
1 & 0 & | & 3 \\
0 & 1 & | & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
3 & -2 & | & (3) \\
7 & 4 & | & (7) \\
9 & 1 & | & (9)
\end{bmatrix}
\]

\[
\begin{bmatrix}
3 & -2 & | & (3) \\
7 & 4 & | & (7) \\
9 & 1 & | & (9)
\end{bmatrix}
\]
6-7 Skills Practice

Using Matrices to Solve Systems of Equations

Write an augmented matrix for each system of equations.

1. \(8x - y = 1\)  
\(x + 2y = -4\)

2. \(5x - 2y = 12\)  
\(2x + y = 8\)

3. \(-2x + 5y = 4\)  
\(4y = 8\)

4. \(-3x + 4y = 22\)  
\(5x + 2y = 4\)

5. \(2x - 3y = 6\)  
\(3x - y = 5\)  
\(3x = 12\)

6. \([-3 \ 4 \ 22\]  
\([1 \ 2 \ 4]\)

7. \([-2 \ -3 \ 6]\)

8. \(-x + 5y = 0\)  
\(8x - 10y = -16\)

9. \(2x = 6\)  
\(3x + 2y = 12\)  
\(3x + 2y = 6\)  
\(x + 4y = 11\)

10. \(2x - y = -2\)  
\(3x + y = 17\)

11. \(x + 4y = 19\)  
\((-1, 5)\)

12. \(-3x - 2y = -7\)  
\(-x + 3y = -11\)

13. \(5x - 2y = 20\)  
\(-x + y = -4\)

14. \(-2x - 4y = 2\)  
\(7x = 14\)  
\(-2x + 2y = -8\)

15. \(9x + y = 6\)

16. \(3x - 2y = 36\)  
\(17. 2x - y = 5\)  
\(18. 4x + y = -13\)

19. \(x + 2y = 0\)  
\(x + y = -5\)

20. \(2x - 5y = 21\)

Use an augmented matrix to solve each system of equations.

21. \(2x - y = -2\)

22. \(x + y = 7\)

23. \((3, 8)\)

24. \((-1, 5)\)

25. \((2, -3)\)

26. \((4, 0)\)

27. \((2, -3)\)

28. \((1, -3)\)

29. \((0, -3)\)

30. \((-2, -5)\)

31. \(5x + 3y = 31.50\)  
\(2x + 6y = 27.00\)

32. \((5, 3, 31.50)\)

33. \((2, 6, 27.00)\)

34. \((0, 4.50)\)

35. \(2x + 3y = -3\)  
\(-2x + 3y = -3\)

36. \(2x + 3y = -6\)  
\(x - 4y = 14\)

37. \(16. COMMUTER RAIL\)  
The cost of a commuter rail ticket varies with the distance traveled. This month, Marcelo bought 5 round-trip tickets to visit his grandmother and 3 round-trip tickets to his friend’s house for $31.50. Last month, Marcelo bought 2 round-trip tickets to visit his grandmother and 6 round-trip tickets to visit his friend’s house for $27.00.

a. Write a system of linear equations to represent the situations.

b. Write the augmented matrix.

c. What is the cost of each type of ticket?

round-trip ticket to his grandmother’s: $4.50; round-trip ticket to his friend’s: $3.00
4. During the 2006–2007 season, the New York Jets scored a total of 59 times by getting touchdowns or kicking field goals, for a combined total of 292 points. If each touchdown is worth 6 points and each field goal is worth 3 points, write an augmented matrix to model the situation. Then find the number of touchdowns scored by the Jets.

5. The design plans for a swimming pool call for a cement walkway of width $x$ feet to surround the entire pool. The pool will be $y$ feet long and $z$ feet wide. The perimeter of the pool is 560 feet, and the outer perimeter of the walkway is 600 feet.

4. **FOOTBALL** During the 2006–2007 season, the New York Jets scored a total of 59 times by getting touchdowns or kicking field goals, for a combined total of 292 points. If each touchdown is worth 6 points and each field goal is worth 3 points, write an augmented matrix to model the situation. Then find the number of touchdowns scored by the Jets.

5. **POOLS** The design plans for a swimming pool call for a cement walkway of width $x$ feet to surround the entire pool. The pool will be $y$ feet long and $z$ feet wide. The perimeter of the pool is 560 feet, and the outer perimeter of the walkway is 600 feet.

a. Write a system of linear equations to model the situation.

$$32x + 2y = 560$$
$$40x + 2y = 600$$

b. Write the augmented matrix.

$$\begin{bmatrix} 32 & 2 & 560 \\ 40 & 2 & 600 \end{bmatrix}$$

c. What are the dimensions of the pool? 200 feet by 80 feet.
**6-8 Study Guide and Intervention**

**Systems of Inequalities**

The solution of a system of inequalities is the set of all ordered pairs that satisfy both inequalities. If you graph the inequalities in the same coordinate plane, the solution is the region where the graphs overlap.

**Example 1**

Solve the system of inequalities by graphing.

\[ y > x + 2 \]
\[ y \leq -2x - 1 \]

The solution includes the ordered pairs in the intersection of the graphs. This region is shaded at the right. The graphs of \( y = x + 2 \) and \( y = -2x - 1 \) are boundaries of this region. The graph of \( y = x + 2 \) is dashed and is not included in the graph of \( y > x + 2 \).

**Example 2**

Solve the system of inequalities by graphing.

\[ x + y > 4 \]
\[ x + y < -1 \]

The graphs of \( x + y = 4 \) and \( x + y = -1 \) are parallel. Because the two regions have no points in common, the system of inequalities has no solution.

**Exercises**

Solve each system of inequalities by graphing.

1. \[ y > -1 \]
   \[ x < 0 \]

2. \[ y > -2x + 2 \]
   \[ y \leq x + 1 \]

3. \[ y < x + 1 \]
   \[ 3x + 4y \geq 12 \]

4. \[ 2x + y \geq 1 \]
   \[ x - y \leq -2 \]

5. \[ y \leq 2x + 3 \]
   \[ y \geq -1 + 2x \]

6. \[ 3x - 2y < 6 \]
   \[ y > -x + 1 \]

**Real-World Problems**

In real-world problems, sometimes only whole numbers make sense for the solution, and often only positive values of \( x \) and \( y \) make sense.

**Example**

**BUSINESS**

AAA Gem Company produces necklaces and bracelets. In a 40-hour week, the company has 400 goms to use. A necklace requires 40 goms and a bracelet requires 10 goms. It takes 2 hours to produce a necklace and a bracelet requires one hour. How many of each type can be produced in a week?

Let \( n \) = the number of necklaces that will be produced and \( b \) = the number of bracelets that will be produced. Neither \( n \) or \( b \) can be a negative number, so the following system of inequalities represents the conditions of the problems.

\[ n \geq 0 \]
\[ b \geq 0 \]
\[ b + 2n \leq 40 \]
\[ 10b + 4n \leq 400 \]

The solution is the set ordered pairs in the intersection of the graphs. This region is shaded at the right. Only whole-number solutions, such as \((5, 20)\), make sense in this problem.

**Exercises**

For each exercise, graph the solution set. List three possible solutions to the problem.

1. **HEALTH**
   Mr. Flowers is on a restricted diet that allows him to have between 1600 and 2000 Calories per day. His daily fat intake is restricted to between 45 and 55 grams. What daily Calorie and fat intakes are acceptable?

2. **RECREATION**
   Maria had $150 in gift certificates to use at a record store. She bought fewer than 20 recordings. Each tape cost $5.95 and each CD cost $8.95. How many of each type of recording might she have bought?

Sample answers: 1600 Calories, 45 fat grams; 1800 Calories, 50 fat grams; 2000 Calories, 55 fat grams

Sample answers: 10 tapes, 9 CDs; 0 tapes, 16 CDs; 14 tapes, 5 CDs
6-8 Skills Practice

**Systems of Inequalities**

Solve each system of inequalities by graphing.

1. \[ \begin{align*} x &> -1 \\ y &\leq -3 \end{align*} \]

2. \[ \begin{align*} y &> 2 \\ x &< -2 \end{align*} \]

3. \[ \begin{align*} y &> x + 3 \\ y &\leq -1 \end{align*} \]

4. \[ \begin{align*} x &< 2 \\ y - x &\leq 2 \end{align*} \]

5. \[ \begin{align*} x + y &\leq -1 \\ x + y &\geq 3 \end{align*} \]

6. \[ \begin{align*} y - x &> 4 \\ x + y &> 2 \end{align*} \]

7. \[ \begin{align*} y &> x + 1 \\ y &\geq -x + 1 \end{align*} \]

8. \[ \begin{align*} y &\geq -x + 2 \\ y &< 2x - 2 \end{align*} \]

9. \[ \begin{align*} y &< 2x + 4 \\ y &\geq x + 1 \end{align*} \]

Write a system of inequalities for each graph.

10. \[ \begin{align*} y &\leq x + 2, y \geq x - 3 \end{align*} \]

11. \[ \begin{align*} y &> -x, y > x \end{align*} \]

12. \[ \begin{align*} y &\geq x + 1, y < 1 \end{align*} \]

6-8 Practice

**Systems of Inequalities**

Solve each system of inequalities by graphing.

1. \[ \begin{align*} y &> x - 2 \\ y &\leq x \end{align*} \]

2. \[ \begin{align*} y &\geq x + 2 \\ y &< 2x + 3 \end{align*} \]

3. \[ \begin{align*} x + y &\geq 1 \\ x + 2y &< 1 \end{align*} \]

4. \[ \begin{align*} y &< 2x - 1 \\ y &> 2 - x \end{align*} \]

5. \[ \begin{align*} y &> x - 4 \\ 2x + y &\leq 2 \end{align*} \]

6. \[ \begin{align*} 2x - y &\geq 2 \\ x - 2y &\geq 2 \end{align*} \]

7. **FITNESS** Diego started an exercise program in which each week he works out at the gym between 4.5 and 6 hours and walks between 9 and 12 miles.

   a. Make a graph to show the number of hours Diego works out at the gym and the number of miles he walks per week.

   b. List three possible combinations of working out and walking that meet Diego's goals. Sample answers: gym 5 h, walk 9 mi; gym 6 h, walk 10 mi; gym 5.5 h, walk 11 mi

8. **SOUVENIRS** Emily wants to buy turquoise stones on her trip to New Mexico to give to at least 4 of her friends. The gift shop sells stones for either $4 or $6 per stone. Emily has no more than $30 to spend.

   a. Make a graph showing the numbers of each price of stone Emily can purchase.

   b. List three possible solutions. Sample answer: one $4 stone and four $6 stones; three $4 stones and three $6 stones; five $4 stones and one $6 stone
### 6-8 Word Problem Practice

**Systems of Inequalities**

1. **PETS** Renée's Pet Store never has more than a combined total of 20 cats and dogs and never more than 8 cats. This is represented by the inequalities \( x \leq 8 \) and \( x + y \leq 20 \). Solve the system of inequalities by graphing.

2. **WAGES** The minimum wage for one group of workers in Texas is $7.25 per hour effective Sept. 1, 2008. The graph below shows the possible weekly wages for a person who makes at least minimum wages and works at most 40 hours. Write the system of inequalities for the graph.

3. **FUND RAISING** The Camp Courage Club plans to sell tins of popcorn and peanuts as a fundraiser. The Club members have $900 to spend on products to sell ... costs $3 and each tin of peanuts costs $4. Write a system of equations to represent the conditions of this problem.

4. **BUSINESS** For maximum efficiency, a factory must have at least 100 workers, but no more than 200 workers on a shift. The factory also must manufacture at least 30 units per worker.

   a. Let \( x \) be the number of workers and \( y \) be the number of units. Write four inequalities expressing the conditions in the problem given above.

   \[
   x \leq 200; \\
   x \geq 100; \\
   y \geq 3000; \\
   y \geq 30x
   \]

   b. Graph the systems of inequalities.

   c. List at least three possible solutions. Sample answer: (110, 3410), (150, 5100), (180, 6300)

### 6-8 Enrichment

**Describing Regions**

The shaded region inside the triangle can be described with a system of three inequalities.

\[
\begin{align*}
y &< -x + 1 \\
y &> 1 \\
3x + 4y &< 900
\end{align*}
\]

Write systems of inequalities to describe each region. You may first need to divide a region into triangles or quadrilaterals.

1. \[
\begin{align*}
y &\leq \frac{5}{2}x + 12 \\
y &\leq -\frac{5}{2}x + \frac{19}{2} \\
y &\leq 2 \\
y &\geq -3
\end{align*}
\]

2. \[
\begin{align*}
y &\geq -x - 2 \\
y &\leq x + 2 \\
y &\geq x - 2 \\
y &\leq -x + 2 \\
y &\leq 5 \\
y &\geq -5
\end{align*}
\]

3. \[
\begin{align*}
top: y &\leq \frac{3}{2}x + 7, y \leq -\frac{3}{2}x + 7, y \geq 4 \\
middle: y &\leq 4, y \geq 0, y \geq -\frac{4}{3}x - 4, \\
y &\geq \frac{3}{2}x - 4 \\
bottom left: y &\leq 4x + 12, y \geq \frac{1}{2}x - 2, \\
&y \leq 0, x \leq 0 \\
bottom right: y &\leq -4x + 12, \\
&y \geq -\frac{1}{2}x - 2, y \leq 0, x \geq 0
\end{align*}
\]
Graphing Calculator Activity

Shading Absolute Value Inequalities

Absolute value inequalities of the form \( y \leq a |x - h| \pm k \) can be graphed using the SHADE command or by using the shading feature in the Y= screen.

**Example 1**

Graph \( y \leq 2 |x - 3| + 2 \).

**Method 1** SHADE command

Enter the boundary equation \( y = 2 |x - 3| + 2 \) into Y1. From the home screen, use SHADE to shade the region under the boundary equation because the inequality uses \( < \). Use Ymin as the lower boundary and the boundary equation, Y1, as the upper boundary.

**Method 2** Y=

On the Y= screen, move the cursor to the far left and repeatedly press ENTER to toggle through the graph choices. Be sure to select the option to shade below. Then enter the boundary equation and graph.

The same techniques can be used to solve a system of absolute value inequalities.

**Example 2**

Solve the system \( y \geq |x + 5| - 4 \) and \( y \leq |x - 6| + 1 \).

Enter \( y = |x + 5| - 4 \) in Y1 and set the graph to shade above. Then enter \( y = |x - 6| + 1 \) in Y2 and set the graph to shade below. Graph the inequalities. Notice that a different pattern is used for the second inequality. The region where the two patterns overlap is the solution to the system.

**Exercises**

Solve each system of inequalities by graphing.

1. \( y \leq 2 |x - 4| + 1 \)
2. \( y \geq \frac{1}{2} |x - 3| + 2 \)
3. \( y \leq 4 |x - 4| + 3 \)
4. \( y \geq \frac{1}{3} |x + 1| + 1 \)

**Spreadsheet Activity**

Systems of Inequalities

**Example**

TopSport Shoe Company has a total of 9600 minutes of machine time each week to cut the materials for the two types of athletic shoes they make. There are a total of 28,000 minutes of worker time per week for assembly. It takes 3 minutes to cut and 12 minutes to assemble a pair of Runners and 2 minutes to cut and 10 minutes to assemble a pair of Flyers. Is it possible for the company to make 1200 pairs of Runners and 1400 pairs of Flyers in a week?

**Step 1**

Represent the situation using a system of inequalities. Let \( r \) represent the number of Runners and \( f \) represent the number of Flyers.

\[ 3r + 12f \leq 9600 \]
\[ 12r + 10f \leq 28,000 \]

**Step 2**

Columns A and B contain the values of \( r \) and \( f \). Columns C and D contain the formulas for the inequalities. The formulas will return TRUE or FALSE. If both inequalities are true for an ordered pair, then the ordered pair is a solution to the system of inequalities.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>1100</td>
<td>Flyers yes</td>
<td>Flyers yes</td>
</tr>
<tr>
<td>2000</td>
<td>500</td>
<td>Flyers no</td>
<td>Flyers yes</td>
</tr>
<tr>
<td>4000</td>
<td>1400</td>
<td>Flyers yes</td>
<td>Flyers yes</td>
</tr>
<tr>
<td>1200</td>
<td>1000</td>
<td>Runners 1200</td>
<td>Runners 1000</td>
</tr>
<tr>
<td>1400</td>
<td>1800</td>
<td>Flyers yes</td>
<td>Flyers no</td>
</tr>
</tbody>
</table>

Since one of the inequalities is false for (1200, 1400), the ordered pair is not part of the solution set. The company cannot make 1200 pairs of Runners and 1400 pairs of Flyers in a week.

**Exercises**

Use the spreadsheet to determine whether TopSport can make the following combinations of shoes.

1. 1000 Runners, 1100 Flyers
2. 1200 Runners, 1000 Flyers
3. 2000 Runners, 500 Flyers
4. 300 Runners, 1500 Flyers
5. 1400 Runners, 1800 Flyers

6. If TopSport can either buy another cutting machine or hire more assemblers, which would make more combinations of shoe production possible? Explain.

Hire more assemblers. Each ordered pair that was not a solution of the system was true for the first inequality. Changing the second inequality may make some of those ordered pairs solutions of the system.
ERROR: undefined
OFFENDING COMMAND:

STACK: