**3 Anticipation Guide**

**Graphing Relations and Functions**

**Step 1 Before you begin Chapter 3**

- Read each statement.
- Decide whether you agree (A) or disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

<table>
<thead>
<tr>
<th>STEP 1 A, D, or NS</th>
<th>Statement</th>
<th>STEP 2 A or D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>The equation $6x + 2xy = 5$ is a linear equation because each variable is to the first power.</td>
<td>D</td>
</tr>
<tr>
<td>2.</td>
<td>The graph of $y = 0$ has more than one $x$-intercept.</td>
<td>A</td>
</tr>
<tr>
<td>3.</td>
<td>The zero of a function is located at the $y$-intercept of the function.</td>
<td>D</td>
</tr>
<tr>
<td>4.</td>
<td>All horizontal lines have an undefined slope.</td>
<td>D</td>
</tr>
<tr>
<td>5.</td>
<td>The slope of a line can be found from any two points on the line.</td>
<td>A</td>
</tr>
<tr>
<td>6.</td>
<td>A direct variation, $y = kx$, will always pass through the origin.</td>
<td>A</td>
</tr>
<tr>
<td>7.</td>
<td>In a direct variation $y = kx$, if $k &lt; 0$ then its graph will slope upward from left to right.</td>
<td>D</td>
</tr>
<tr>
<td>8.</td>
<td>A sequence is arithmetic if the difference between all consecutive terms is the same.</td>
<td>A</td>
</tr>
<tr>
<td>9.</td>
<td>Each number in a sequence is called a factor of that sequence.</td>
<td>D</td>
</tr>
<tr>
<td>10.</td>
<td>Making a conclusion based on a pattern of examples is called inductive reasoning.</td>
<td>A</td>
</tr>
</tbody>
</table>

**Step 2 After you complete Chapter 3**

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

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**3-1 Study Guide and Intervention**

**Graphing Linear Equations**

**Identify Linear Equations and Intercepts** A linear equation is an equation that can be written in the form $Ax + By = C$. This is called the standard form of a linear equation.

**Standard Form of a Linear Equation** $Ax + By = C$, where $A$ and $B$ are not both zero, and $A$, $B$, and $C$ are integers with GCF of 1.

**Example 1**

Determine whether $y = 6 - 3x$ is a linear equation. Write the equation in standard form.

**Solution**

First rewrite the equation so both variables are on the same side of the equation.

$y + 3x = 6 - 3x + 3x$

$3x + y = 6$

The equation is now in standard form, with $A = 3$, $B = 1$ and $C = 6$. This is a linear equation.

**Example 2**

Determine whether $3y + x = 4 + 2x$ is a linear equation. Write the equation in standard form.

Since the term $3y$ has two variables, the equation cannot be written in the form $Ax + By = C$. Therefore, this is not a linear equation.

**Exercises**

Determine whether each equation is a linear equation. Write yes or no. If yes, write the equation in standard form.

1. $2x + 4y = 0$
   - yes; $2x - 4y = 0$
2. $6 + y = 8$
   - yes; $y = 2$
3. $4x - 2y = -1$
   - yes; $4x - 2y = -1$
4. $3xy + 8 = 4y$
   - no
5. $3x - 4 = 12$
   - yes; $3x = 16$
6. $y = x^2 + 7$
   - no
7. $y - 4x = 9$
   - yes; $4x - y = 9$
   - yes; $x = -8$
8. $8. x + 8 = 0$
   - yes; $x = 8$
9. $-2x + 3 = 4y$
   - yes; $2x + 4y = 3$
10. $2 + \frac{1}{2}x = y$
    - yes; $x = 12 - 4x$
   - yes; $16x + y = 48$
11. $1.5y = 12 - 4x$
    - no
12. $3xy - y = 8$
    - yes; $6x + 4y = 3$
13. $14. xy - 2 = 8$
    - no
15. $15. 6x - 2y = 8 + y$
    - yes; $6x - 3y = 8$
16. $16. \frac{1}{2}x - 12y = 1$
    - yes; $x = 12y$
   - yes; $x + 4y = 0$
17. $17. 3 + x + x^2 = 0$
    - no
18. $18. x^2 = 2xy$
    - no
Graph Linear Equations: The graph of a linear equation represents all the solutions of the equation. An x-coordinate of the point at which a graph of an equation crosses the x-axis is called an x-intercept. A y-coordinate of the point at which a graph crosses the y-axis is called a y-intercept.

Example 1: Graph the equation \(3x + 2y = 6\) by making a table.

To find the x-intercept, let \(y = 0\) and solve for \(x\). The equation becomes \(3x = 6\), so the x-intercept is 2. The graph intersects the x-axis at (2, 0).

To find the y-intercept, let \(x = 0\) and solve for \(y\). The equation becomes \(2y = 6\), so the y-intercept is 3. The graph intersects the y-axis at (0, 3).

Plot the points (2, 0) and (0, 3) and draw the line through them.

Example 2: Graph the equation \(y = 2x + 1\) by making a table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(-2)</th>
<th>(-1)</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2x + 1)</td>
<td>(-3)</td>
<td>(-1)</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>((x, y))</td>
<td>((-2, -3))</td>
<td>((-1, -1))</td>
<td>((0, 1))</td>
<td>((1, 3))</td>
<td>((2, 5))</td>
</tr>
</tbody>
</table>

Solve the equation for \(y\), \(y = 2x + 1\). Add 2 to each side, \(y = 2x + 1\). Select five values for the domain and make a table. Then graph the ordered pairs and draw a line through the points.

Exercises:

Graph each equation by using the x- and y-intercepts.

1. \(2x + y = -2\)
2. \(3x - 6y = -3\)
3. \(-2x + y = -2\)
4. \(y = 2x\)
5. \(x - y = -1\)
6. \(x + 2y = 4\)

Graph each equation by making a table.

7. \(y = 2x - 3\)
8. \(x - y = 1\)
9. \(2x + y = 3\)

Graph each equation by using the x- and y-intercepts.

10. \(x - y = 2\)
11. \(x - y = -2\)
12. \(x - y = 2\), \(y = 2\)

Find the x- and y-intercepts of each linear function.

13. \(y = 4\)
14. \(y = 3x\)
15. \(y = x + 4\)
16. \(x - y = 3\)
17. \(10x = -5y\)
18. \(4x = 2y + 6\)
The equation \( y = 1000x - 5000 \) represents the profit in dollars is \( y \). The first values date of operation is when time is zero. However, preparation for opening the business began 3 months earlier with the purchase of equipment and supplies.

Graph the linear function for \( 3 \) to \( 8 \).

The height of a woman whose radius bone is 25 centimeters long is her height in centimeters and \( r \) is the length of her radius.

Graph each equation. 7.

1. \( 4y + 2y = 9 \) no
2. \( 8x - 3y = 6 - 4x \) yes; \( 4x - y = 2 \); \( x = \frac{1}{2}, y = -2 \)
3. \( 7x + y + 3 = y \) yes; \( 7x = -3 \); \( x = -\frac{3}{7}, y \) none
4. \( 5 - 2y = 3x \) yes; \( 3x + 2y = 5 \); \( x = \frac{5}{3}, y = \frac{5}{2} \)

Graph each equation.
8. \( 5x - 2y = 7 \)
9. \( 1.5x + 3y = 9 \)

10. COMMUNICATIONS A telephone company charges $4.95 per month for long distance calls plus $0.05 per minute. The monthly cost of long distance calls can be described by the equation \( c = 0.05m + 4.95 \), where \( m \) is the number of minutes.

a. Find the \( y \)-intercept of the graph of the equation. \((0, 4.95)\)

b. Graph the equation.

c. If you talk 140 minutes, what is the monthly cost? $11.95

11. MARINE BIOLOGY Killer whales usually swim at a rate of 3.2–9.7 kilometers per hour, though they can travel up to 48.4 kilometers per hour. Suppose a migrating killer whale is swimming at an average rate of 4.5 kilometers per hour. The distance of the whale has traveled in \( r \) hours can be predicted by the equation \( d = 4.5t \).

a. Graph the equation.

b. Use the graph to predict the time it takes the killer whale to travel 30 kilometers. \( \text{between 6 h and 7 h} \)

c. Use the function to find the approximate height of a woman whose radius bone is 25 centimeters long. 165 cm
3-1 Translating Linear Graphs

Linear graphs can be translated on the coordinate plane. This means that the graph moves up, down, right, or left without changing its direction. Translating the graphs up or down affects the y-coordinate for a given x-value. Translating the graph right or left affects the x-coordinate for a given y-value.

Example
Translate the graph of \( y = 2x + 2 \), 3 units up.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

Add 3 to each y-value.

Exercises
Graph the function and the translation on the same coordinate plane.

1. \( y = x + 4 \), 3 units down
2. \( y = 2x - 2 \), 2 units left
3. \( y = -2x + 1 \), 1 unit right
4. \( y = -y - 3 \), 2 units up

Example
An internet retailer charges $1.99 per order plus $0.99 per item to ship books and CDs. Graph the equation \( y = 1.99 + 0.99x \), where \( x \) is the number of items ordered and \( y \) is the shipping cost.

Step 1 Use column A for the numbers of items and column B for the shipping costs.

Step 2 Create a graph from the data. Select the data in columns A and B and select Chart from the Insert menu. Select an XY (Scatter) chart to show the data points connected with line segments.

Exercises
1. A photo printer offers a subscription for digital photo finishing. The subscription costs $4.99 per month. Each standard size photo a subscriber prints costs $0.19. Use a spreadsheet to graph the equation \( y = 4.99 + 0.19x \), where \( x \) is the number of photos printed and \( y \) is the total monthly cost. See students' work.
2. A long distance service plan includes a $8.95 per month fee plus $0.05 per minute of calls. Use a spreadsheet to graph the equation \( y = 8.95 + 0.05x \), where \( x \) is the number of minutes of calls and \( y \) is the total monthly cost. See students' work.
Solving Linear Equations by Graphing

You can solve an equation by graphing the related function. The solution of the equation is the x-intercept of the function.

Example

Solve the equation $2x - 2 = -4$ by graphing.

First set the equation equal to 0. Then replace 0 with $f(x) = 2x + 2$.

\[ \begin{align*}
2x - 2 &= -4 & \text{Original equation} \\
2x - 2 + 4 &= -4 + 4 & \text{Add 4 to each side.} \\
x &= 0 & \text{Simplify.} \\
f(x) &= 2x + 2 & \text{Replace 0 with } f(x).
\end{align*} \]

To graph the function, make a table. Graph the ordered pairs.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = 2x + 2$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>$-1$</td>
<td>$(-1, 1)$</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>$(0, 2)$</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>$(1, 4)$</td>
</tr>
</tbody>
</table>

The graph intersects the x-axis at $(1, 0)$.

The solution to the equation is $x = -1$.

Exercises

Solve each equation.

1. $3x - 3 = 0$
   \[
   \begin{align*}
   3x - 3 &= 0 \\
   3x &= 3 \\
   x &= 1
   \end{align*}
   \]

2. $-2x + 1 = 5 - 2x$
   \[
   \begin{align*}
   -2x + 1 &= 5 - 2x \\
   -2x + 1 - (-2x) &= 5 - 2x - (-2x) \\
   1 &= 5 \\
   \text{No solution}
   \end{align*}
   \]

3. $-x + 4 = 0$
   \[
   \begin{align*}
   -x + 4 &= 0 \\
   -x &= -4 \\
   x &= 4
   \end{align*}
   \]

4. $0 = 4x - 1$
   \[
   \begin{align*}
   4x - 1 &= 0 \\
   4x &= 1 \\
   x &= \frac{1}{4}
   \end{align*}
   \]

5. $5x - 1 = 5x$
   \[
   \begin{align*}
   5x - 1 &= 5x \\
   5x - 1 - 5x &= 5x - 5x \\
   -1 &= 0 \\
   \text{No solution}
   \end{align*}
   \]

6. $-3x + 1 = 0$
   \[
   \begin{align*}
   -3x + 1 &= 0 \\
   -3x &= -1 \\
   x &= \frac{1}{3}
   \end{align*}
   \]

### Example (continued)

Solving Linear Equations by Graphing

Estimate Solutions by Graphing

Sometimes graphing does not provide an exact solution, but only an estimate. In these cases, solve algebraically to find the exact solution.

Exercise

WALKING You and your cousin decide to walk the 7-mile trail at the state park to the ranger station. The function $d = 7 - 3.2t$ represents your distance $d$ from the ranger station after $t$ hours. Find the zero of this function.

Describe what this value means in this context.

Make a table of values to graph the function.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$d = 7 - 3.2t$</th>
<th>$(t, d)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$7 - 3.2(0)$</td>
<td>$(0, 7)$</td>
</tr>
<tr>
<td>1</td>
<td>$7 - 3.2(1)$</td>
<td>$(1, 3.8)$</td>
</tr>
<tr>
<td>2</td>
<td>$7 - 3.2(2)$</td>
<td>$(2, 0.6)$</td>
</tr>
</tbody>
</table>

The graph intersects the t-axis at $(2, 0)$.

The solution to the equation is $t = 2$.

You can check your estimate by solving the equation algebraically.

Exercises

1. MUSIC Jessica wants to record her favorite songs to one CD. The function $C = 80 - 3.22n$ represents the recording time $C$ available after $n$ songs are recorded. Find the zero of this function. Describe what this value means in this context.

   just under 25; only 24 songs can be recorded on one CD

2. GIFT CARDS Enrique uses a gift card to buy coffee at a coffee shop. The initial value of the gift card is $20. The function $n = 20 - 2.75c$ represents the amount of money still left on the gift card $n$ after purchasing $c$ cups of coffee. Find the zero of this function. Describe what this value means in this context.

   just over 7; Enrique can buy 7 cups of coffee with the gift card.


**3-2 Skills Practice**

**Solving Linear Equations by Graphing**

Solve each equation.

1. \(2x - 5 = -3 + 2x\)

   - Graph shows two lines intersecting. No solution.

2. \(-3x + 2 = 0\)

   - Graph shows two lines intersecting at \(x = \frac{2}{3}\).

3. \(3x + 2 = 3x - 1\)

   - Graph shows two vertical lines, no solution.

4. \(4x - 1 = 4x + 2\)

   - Graph shows two horizontal lines, no solution.

5. \(5x - 1 = 0\)

   - Graph shows two lines intersecting at \(x = \frac{1}{5}\).

6. \(0 = 5x + 3\)

   - Graph shows a line parallel to the x-axis, no solution.

7. \(0 = -2x + 4\)

   - Graph shows a line parallel to the x-axis, no solution.

8. \(-3x + 8 = 5 - 3x\)

   - Graph shows two lines intersecting, no solution.

9. \(-x + 1 = 0\)

   - Graph shows two lines intersecting at \(x = 1\).

10. **GIFT CARDS**
    You receive a gift card for trading cards from a local store. The function \(d = 20 - 1.95c\) represents the remaining dollars \(d\) on the gift card after obtaining \(c\) packages of cards. Find the zero of this function. Describe what this value means in this context.

    - Graph shows a line intersecting the x-axis at \(c = 10.26\). Approximate value: \(\approx 10.26\); you can purchase 10 packages of trading cards with the gift card.

**3-2 Practice**

**Solving Linear Equations by Graphing**

Solve each equation.

1. \(\frac{3}{2}x - 2 = 0\)

2. \(-3x + 2 = -1\)

3. \(4x - 2 = -2\)

4. \(\frac{1}{3}x + 2 = \frac{1}{3}x - 1\)

5. \(\frac{2}{3}x + 4 = 3\)

6. \(\frac{3}{4}x + 1 = \frac{3}{4}x - 7\)

7. \(13x + 2 = 11x - 1\)

8. \(-9x - 3 = -4x - 3\)

9. \(-\frac{1}{5}x + 2 = \frac{2}{3}x - 1\)

10. **DISTANCE**
    A bus is driving at 60 miles per hour toward a bus station that is 250 miles away. The function \(d = 250 - 60t\) represents the distance \(d\) from the bus station the bus is \(t\) hours after it has started driving. Find the zero of this function. Describe what this value means in this context.

    - Graph shows a line intersecting the x-axis at \(t = 4.17\). Approximate value: \(\approx 4.17\) hr; the bus will arrive at the station in approximately 4.17 hours.
3-2 Word Problem Practice
Solving Linear Equations by Graphing

1. **PET CARE** You buy a 6.3-pound bag of dry cat food for your cat. The function \( c = 6.3 - 0.25p \) represents the amount of cat food \( c \) remaining in the bag when the cat is fed the same amount each day for \( p \) days. Find the zero of this function. Describe what this value means in this context.

2. **SAVINGS** Jessica is saving for college using a direct deposit from her paycheck into a savings account. The function \( m = 30.45 - 52.50t \) represents the amount of money \( m \) left in the savings account after \( t \) weeks. Find the zero of this function. Describe what this value means in this context.

3. **FINANCE** Michael borrows $100 from his dad. The function \( v = 100 - 4.75p \) represents the outstanding balance \( v \) after \( p \) weekly payments. Find the zero of this function. Describe what this value means in this context.

4. **BAKE SALE** Ashley has $15 in the Pep Club treasury to pay for supplies for a chocolate chip cookie bake sale. The function \( d = 15 - 0.08c \) represents the dollars \( d \) left in the club treasury after making \( c \) cookies. Find the zero of this function. Describe what this value means in this context.

5. **DENTAL HYGIENE** You are packing your suitcase to go away to a 14-day summer camp. The store carries three sizes of tubes of toothpaste.

<table>
<thead>
<tr>
<th>Tube</th>
<th>Size (ounces)</th>
<th>Size (grams)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.75</td>
<td>21.26</td>
</tr>
<tr>
<td>B</td>
<td>0.9</td>
<td>25.52</td>
</tr>
<tr>
<td>C</td>
<td>3.0</td>
<td>85.04</td>
</tr>
</tbody>
</table>

Source: National Academy of Sciences

- **Rule:**
  \[ f(x) = 2c + 1 \]
  \[ f(1) = 2(1) + 1 = 3 \]
  \[ f(2) = 2(2) + 1 = 5 \]
  \[ f(-3) = 2(-3) + 1 = -5 \]

Suppose we have three sets A, B, and C and two functions described as shown below:

- **Rule:**
  \[ g(y) = 3y - 4 \]
  \[ g(2) = 3(2) - 4 = 2 \]
  \[ g(0) = 3(0) - 4 = -4 \]

Let’s find a rule that will match elements of set A with elements of set C without finding any elements in set B. In other words, let’s find a rule for the composite function \( g(f(x)) \).

Since \( f(x) = 2x + 1 \) and \( g(f(x)) = g(2x + 1) \).

Since \( g(y) = 3y - 4 \), \( g(2x + 1) = 3(2x + 1) - 4 = 2x + 3 \). Therefore, \( g(2x + 1) = 2x + 3 \).

Find a rule for the composite function \( g(f(x)) \).

- **Rule:**
  \[ g(f(x)) = 4x^2 + 4 \]
  \[ g(f(x)) = 4x^2 + 4 \]

5. Is it always the case that \( g(f(x)) = f(g(x)) \)? Justify your answer.

No. For example, in Exercise 1,

\[ f(g(x)) = f(2x + 1) = 3(2x + 1) = 6x + 3, \text{ not } 6x + 1. \]
**Rate of Change and Slope**

The rate of change tells, on average, how a quantity is changing over time. The slope of a line is the ratio of change in the y-coordinates (rise) to the change in the x-coordinates (run) as you move in the positive direction.

**Example**

Population Growth in China

<table>
<thead>
<tr>
<th>Year</th>
<th>Population Growth (in billions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950</td>
<td>2.5</td>
</tr>
<tr>
<td>1975</td>
<td>4.5</td>
</tr>
<tr>
<td>2000</td>
<td>7.5</td>
</tr>
<tr>
<td>2025</td>
<td>10.5</td>
</tr>
</tbody>
</table>

**Exercises**

1. **LONGEVITY** The graph shows the predicted life expectancy for men and women born in a given year.
   a. Find the rates of change for men from 2000–2025 and 2025–2050. 0.16/yr, 0.12/yr.
   b. Find the rates of change for men from 2000–2025 and 2025–2050. 0.16/yr, 0.12/yr.

2. **Predicting Life Expectancy** The graph shows the predicted life expectancy for men and women born in a given year.
   a. Find the rates of change for women from 2000–2025 and 2025–2050. 0.16/yr, 0.12/yr.
   b. Explain the meaning of your results in Exercises 1 and 2. Both men and women increased their life expectancy at the same rates.

3. **What pattern do you see in the increase with each 25-year period?** While life expectancy increases, it does not increase at a constant rate.

4. **Make a prediction for the life expectancy for 2050–2075.** Explain how you arrived at your prediction. Sample answer: 89 for women and 83 for men; the decrease in rate from 2000–2025 to 2025–2050 is 0.04/yr. If the decrease in the rate remains the same, the change of rate for 2050–2075 might be 0.08/yr and 25(0.08) = 2 years of increase over the 25-year span.

**Exercises (continued)**

Find the slope of the line that passes through (3, 5) and (4, –2).

Let \((x_1, y_1) = (3, 5)\) and \((x_2, y_2) = (4, –2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 5}{4 - 3} = \frac{-7}{1} = -7
\]

**Example 2** Find the value of \(r\) so that the line through (10, 3) and (3, 4) has a slope of \(-\frac{1}{7}\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1}\]

Let \((x_1, y_1) = (10, 3)\) and \((x_2, y_2) = (3, 4)\).

\[
m = \frac{4 - 3}{3 - 10} = \frac{-1}{7}
\]

**Exercises**

Find the slope of the line that passes through each pair of points.

1. (4, 9), (1, 6) 12. (–4, –1), (–2, –5) 3. (–4, –1), (–4, –5) undefined
2. (2, 1), (8, \(\frac{4}{3}\)) 5. (14, –8), (11, –6) \(-\frac{2}{3}\)
3. (4, –3), (8, –3) 0 6. (4, –3), (8, –3) 0
4. (2, 5), (6, 2) \(-\frac{3}{4}\) 8. (2, 5), (6, 2) \(-\frac{3}{4}\)
5. (14, –8), (11, –6) \(-\frac{2}{3}\) 9. (4, 3.5), (–4, 3.5) 0
6. (6, 8), (r, –2) \(m = 1 = 4\)
7. (1, –2), (6, 2) \(m = \frac{4}{5}\)
8. (6, 8), (r, –2) \(m = 1 = 4\)
9. (2, 5), (6, 2) \(-\frac{3}{4}\)
10. (14, –8), (11, –6) \(-\frac{2}{3}\)
11. (4, –3), (8, –3) 0
12. (2, 5), (6, 2) \(-\frac{3}{4}\)
13. (7, –5), (6, r) \(m = \frac{3}{4}\)
14. (4, r), (7, 1) \(m = \frac{3}{4}\)
15. (7, 5), (r, 9) \(m = \frac{23}{3}\)
3-3 Practice
Rate of Change and Slope

Find the slope of the line that passes through each pair of points.

1.\((0, 1), (2, 4)\)
2.\((1, -2), (3, 5)\)
3.\((-2, 3), (4, 6)\)

4.\((-2, 3), (4, 8)\)
5.\((-6, -1), (6, 1)\)
6.\((5, 2), (5, -2)\)

7.\((x, 3), (x, 5)\)
8.\((-3, 5), (-3, -5)\)
9.\((9, 8), (7, -8)\)

10.\((-5, -8), (-8, 1)\)
11.\((-3, 10), (-3, 7)\)
12.\((17, 18), (18, 17)\)

13.\((-6, -4), (4, 1)\)
14.\((-2, 4), (-2 - 4)\)
15.\((-2, 1), (-8, -2)\)
16.\((-5, -9), (3, -2)\)
17.\((12, 6), (3, -5)\)
18.\((-4, 5), (-8, -5)\)

Find the value of \(x\) so the line that passes through each pair of points has the given slope.

19.\((0, 1), (2, 4)\)
20.\((0, 1), (2, 4)\)
21.\((0, 1), (2, 4)\)
22.\((0, 1), (2, 4)\)

23.\((x, 3), (x, 5)\)
24.\((-2, 3), (4, 6)\)
25.\((x, 3), (x, 5)\)

Find the value of \(x\) so the line that passes through each pair of points has the given slope.

26.\((-2, 3), (4, 6)\)
27.\((-2, 3), (4, 6)\)
28.\((-2, 3), (4, 6)\)
29.\((-2, 3), (4, 6)\)

30.\((-2, 3), (4, 6)\)
31.\((-2, 3), (4, 6)\)
32.\((-2, 3), (4, 6)\)
33.\((-2, 3), (4, 6)\)

34.\((-2, 3), (4, 6)\)
35.\((-2, 3), (4, 6)\)
36.\((-2, 3), (4, 6)\)
37.\((-2, 3), (4, 6)\)

38.\((-2, 3), (4, 6)\)
39.\((-2, 3), (4, 6)\)
40.\((-2, 3), (4, 6)\)
41.\((-2, 3), (4, 6)\)

42.\((-2, 3), (4, 6)\)
43.\((-2, 3), (4, 6)\)
44.\((-2, 3), (4, 6)\)
45.\((-2, 3), (4, 6)\)

46.\((-2, 3), (4, 6)\)
47.\((-2, 3), (4, 6)\)
48.\((-2, 3), (4, 6)\)
49.\((-2, 3), (4, 6)\)

50.\((-2, 3), (4, 6)\)
51.\((-2, 3), (4, 6)\)
52.\((-2, 3), (4, 6)\)
53.\((-2, 3), (4, 6)\)

54.\((-2, 3), (4, 6)\)
55.\((-2, 3), (4, 6)\)
56.\((-2, 3), (4, 6)\)
57.\((-2, 3), (4, 6)\)

58.\((-2, 3), (4, 6)\)
59.\((-2, 3), (4, 6)\)
60.\((-2, 3), (4, 6)\)
61.\((-2, 3), (4, 6)\)

62.\((-2, 3), (4, 6)\)
63.\((-2, 3), (4, 6)\)
64.\((-2, 3), (4, 6)\)
65.\((-2, 3), (4, 6)\)

66.\((-2, 3), (4, 6)\)
67.\((-2, 3), (4, 6)\)
68.\((-2, 3), (4, 6)\)
69.\((-2, 3), (4, 6)\)

70.\((-2, 3), (4, 6)\)
71.\((-2, 3), (4, 6)\)
72.\((-2, 3), (4, 6)\)
73.\((-2, 3), (4, 6)\)

74.\((-2, 3), (4, 6)\)
75.\((-2, 3), (4, 6)\)
76.\((-2, 3), (4, 6)\)
77.\((-2, 3), (4, 6)\)

78.\((-2, 3), (4, 6)\)
79.\((-2, 3), (4, 6)\)
80.\((-2, 3), (4, 6)\)
81.\((-2, 3), (4, 6)\)

82.\((-2, 3), (4, 6)\)
83.\((-2, 3), (4, 6)\)
84.\((-2, 3), (4, 6)\)
85.\((-2, 3), (4, 6)\)

86.\((-2, 3), (4, 6)\)
87.\((-2, 3), (4, 6)\)
88.\((-2, 3), (4, 6)\)
89.\((-2, 3), (4, 6)\)

90.\((-2, 3), (4, 6)\)
91.\((-2, 3), (4, 6)\)
92.\((-2, 3), (4, 6)\)
93.\((-2, 3), (4, 6)\)

94.\((-2, 3), (4, 6)\)
95.\((-2, 3), (4, 6)\)
96.\((-2, 3), (4, 6)\)
97.\((-2, 3), (4, 6)\)

98.\((-2, 3), (4, 6)\)
99.\((-2, 3), (4, 6)\)
100.\((-2, 3), (4, 6)\)
101.\((-2, 3), (4, 6)\)
3-3 Word Problem Practice
Rate of Change and Slope

1. HIGHWAYS Roadway signs such as the one below are used to warn drivers of an upcoming steep down grade that could lead to a dangerous situation. What is the grade, or slope, of the hill described on the sign?

\[ \frac{2}{25} \]

2. AMUSEMENT PARKS The SheiKra roller coaster at Busch Gardens in Tampa, Florida, features a 138-foot vertical drop. What is the slope of the coaster track at this part of the ride? Explain.

The slope is undefined because the drop is vertical.

3. CENSUS The table shows the population density for the state of Texas in various years. Find the average annual rate of change in the population density from 2000 to 2006.

<table>
<thead>
<tr>
<th>Year</th>
<th>People Per Square Mile</th>
</tr>
</thead>
<tbody>
<tr>
<td>1930</td>
<td>22.1</td>
</tr>
<tr>
<td>1950</td>
<td>36.4</td>
</tr>
<tr>
<td>1980</td>
<td>54.3</td>
</tr>
<tr>
<td>2000</td>
<td>79.6</td>
</tr>
<tr>
<td>2006</td>
<td>87.5</td>
</tr>
</tbody>
</table>

Source: Bureau of the Census, U.S. Dept. of Commerce

Increased about 1.32 people per square mile

4. REAL ESTATE The median price of an existing single-family home in the United States was $195,200 in 2004. The median price had risen to $221,900 by 2006. Find the average annual rate of change in median home price from 2004 to 2006.

\[ \frac{13,075}{2} \]

5. COAL EXPORTS The graph shows the annual coal exports from U.S. mines in millions of short tons.

![Coal Export Graph]

a. What was the rate of change in coal exports between 2001 and 2002?

\[-9 \text{ million tons per year or } -\frac{9}{1}\]

b. How does the rate of change in coal exports from 2005 to 2006 compare to that of 2001 to 2002?

In 2005–2006, the rate was 0 compared to \(-\frac{9}{1}\) in 2001–2002.

c. Explain the meaning of the part of the graph with a slope of zero.

The slope indicates that there was no change in the amount of coal exported between 2005 and 2006.

3-3 Enrichment
Treasure Hunt with Slopes

Using the definition of slope, draw segments with the slopes listed below in order. A correct solution will trace the route to the treasure.

1. \(\frac{3}{2}\)
2. \(-\frac{1}{4}\)
3. \(-\frac{2}{5}\)
4. 0
5. 1
6. \(-1\)
7. no slope
8. \(\frac{3}{2}\)
9. \(-\frac{3}{4}\)
10. \(-\frac{1}{3}\)
11. \(-\frac{3}{4}\)
12. 3

Start Here
3-4 Study Guide and Intervention

Direct Variation

Direct Variation Equations A direct variation is described by an equation of the form \( y = kx \), where \( k \neq 0 \). We say that \( y \) varies directly as \( x \). In the equation \( y = kx \), \( k \) is the constant of variation.

Example 1 Name the constant of variation for the equation. Then find the slope of the line that passes through the pair of points.

For \( y = \frac{1}{2}x \), the constant of variation is \( \frac{1}{2} \).

\[
\text{Slope formula} \quad = \frac{y_2 - y_1}{x_2 - x_1} \\
= \frac{1 - 0}{2 - 0} \quad (0, 2), \quad (2, 1) \\
= \frac{1}{2} \quad \text{Simplify.}
\]

The slope is \( \frac{1}{2} \).

Example 2 Suppose \( y \) varies directly as \( x \), and \( y = 30 \) when \( x = 5 \).

a. Write a direct variation equation that relates \( x \) and \( y \).

Find the value of \( k \).

\[
y = kx \quad \text{Direct variation equation} \\
30 = k(5) \quad \text{Replace } y \text{ with } 30 \text{ and } x \text{ with } 5. \\
6 = k \quad \text{Divide each side by } 5.
\]

Therefore, the equation is \( y = 6x \).

b. Use the direct variation equation to find \( x \) when \( y = 18 \).

\[
y = 6x \quad \text{Direct variation equation} \\
18 = 6x \quad \text{Replace } y \text{ with } 18. \\
3 = x \quad \text{Divide each side by } 6.
\]

Therefore, \( x = 3 \) when \( y = 18 \).

Exercises

Name the constant of variation for each equation. Then determine the slope of the line that passes through each pair of points.

1. \( y = \frac{1}{2}x \)

2. \( y = 3x \)

3. \( y = \frac{3}{2}x \)

Suppose \( y \) varies directly as \( x \). Write a direct variation equation that relates \( x \) to \( y \). Then solve.

4. If \( y = 4 \) when \( x = 2 \), find \( y \) when \( x = 16 \). \( y = 2x; 32 \)

5. If \( y = 9 \) when \( x = 3 \), find \( x \) when \( y = 6 \). \( y = 3x; \frac{1}{2} \)

6. If \( y = -4.8 \) when \( x = -1.6 \), find \( x \) when \( y = -24 \). \( y = 3x; -8 \)

7. If \( y = 1 \) when \( x = \frac{1}{3} \), find \( x \) when \( y = \frac{1}{16} \). \( y = 2x; \frac{3}{32} \)

Example 3 TRAVEL A family drove their car 225 miles in 5 hours.

a. Write a direct variation equation to find the distance traveled for any number of hours.

Use given values for \( d \) and \( t \) to find \( r \).

\[
d = rt \quad \text{Original equation} \\
225 = rt \quad \text{Replace } d \text{ with } 225 \text{ and } t \text{ with } 5. \\
45 = r \quad \text{Divide each side by } 5.
\]

Therefore, the direct variation equation is \( d = 45t \).

b. Graph the equation.

The graph of \( d = 45t \) passes through the origin with slope 45.

\[
m = \frac{45}{5} \quad \text{Simplify.} \\
= 9 \quad \text{Slope}
\]

\[
\text{CHECK (5, 225) lies on the graph.}
\]

c. Estimate how many hours it would take the family to drive 360 miles.

\[
d = 45t \quad \text{Original equation} \\
360 = 45t \quad \text{Replace } d \text{ with } 360. \\
t = 8 \quad \text{Divide each side by } 45.
\]

Therefore, it will take 8 hours to drive 360 miles.

Exercises

1. RETAIL The total cost \( C \) of bulk jelly beans is \$4.49 times the number of pounds \( p \).

a. Write a direct variation equation that relates the variables.

\( C = 4.49p \)

b. Graph the equation on the grid at the right.

c. Find the cost of \( \frac{3}{4} \) pound of jelly beans. \$3.37

2. CHEMISTRY Charles’s Law states that, at a constant pressure, volume of a gas \( V \) varies directly as its temperature \( T \). A volume of 4 cubic feet of a certain gas has a temperature of 200 degrees Kelvin.

a. Write a direct variation equation that relates the variables.

\( V = 0.027 \)

b. Graph the equation on the grid at the right.

c. Find the volume of the same gas at 250 degrees Kelvin. \( 5 \text{ ft}^3 \)
3-4 Direct Variation

Skills Practice

Name the constant of variation for each equation. Then determine the slope of the line that passes through each pair of points.

1. \( y = - \frac{1}{3} x \)
2. \( y = -2; -2 \)
3. \( y = \frac{3}{2}; -\frac{3}{2} \)

Graph each equation.

4. \( y = 3x \)
5. \( y = -\frac{3}{4} x \)
6. \( y = \frac{2}{5} x \)

Suppose \( y \) varies directly as \( x \). Write a direct variation equation that relates \( x \) and \( y \). Then solve.

7. If \( y = -8 \) when \( x = -2 \), find \( x \) when \( y = 32 \).
8. If \( y = 45 \) when \( x = 15 \), find \( x \) when \( y = 15 \).
9. If \( y = -4 \) when \( x = 2 \), find \( y \) when \( x = -6 \).
10. If \( y = -9 \) when \( x = 3 \), find \( y \) when \( x = -5 \).
11. If \( y = 4 \) when \( x = 16 \), find \( y \) when \( x = 6 \).
12. If \( y = 12 \) when \( x = 18 \), find \( y \) when \( x = -18 \).

Write a direct variation equation that relates the variables.

Then graph the equation.

13. TRAVEL The total cost \( C \) of gasoline is $3.00 times the number of gallons \( g \).
14. SHIPPING The number of delivered toys \( T \) is 3 times the total number of crates \( c \).

<table>
<thead>
<tr>
<th>Gasoline Cost</th>
<th>Toys Shipped</th>
<th>T = 3c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>Gallons</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>40</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>60</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>80</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>100</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

Answers

1. \( y = -\frac{3}{4} x \)
2. \( y = \frac{2}{5} x \)
3. \( y = -\frac{3}{2} x \)

Suppose \( y \) varies directly as \( x \). Write a direct variation equation that relates \( x \) and \( y \). Then solve.

7. If \( y = 7.5 \) when \( x = 0.5 \), find \( y \) when \( x = -0.3 \). \( y = 15x; -4.5 \)
8. If \( y = 40 \) when \( x = 32 \), find \( y \) when \( x = 100 \). \( y = 2.5x; 40 \)
9. If \( y = 3 \) when \( x = 24 \), find \( y \) when \( x = 12 \). \( y = \frac{1}{3} x; 9 \)

Write a direct variation equation that relates the variables. Then graph the equation.

10. MEASURE The width \( W \) of a rectangle is two thirds of the length \( l \).
11. TICKETS The total cost \( C \) of tickets is $4.50 times the number of tickets \( t \).

12. PRODUCE The cost of bananas varies directly with their weight. Miguel bought \( \frac{3}{2} \) pounds of bananas for $1.12. Write an equation that relates the cost of the bananas to their weight. Then find the cost of \( \frac{3}{4} \) pounds of bananas. \( C = 0.32p; $1.36 \)
Chapter 3

Lesson 3-4

1. **ENGINES**
The engine of a chainsaw requires a mixture of engine oil and gasoline. According to the directions, oil and gasoline should be mixed as shown in the graph below. What is the constant of variation for the line graphed?

2. **RACING**
In 2007, English driver Lewis Hamilton won the United States Grand Prix at the Indianapolis Motor Speedway. His speed during the race averaged 125.145 miles per hour. Write a direct variation equation for the distance d that Hamilton drove in h hours at that speed.

3. **CURRENCY**
The exchange rate from U.S. dollars to British pound sterling (£) was approximately $2.07 to £1 in 2007. Write and solve a direct variation equation to determine how many pounds sterling you would receive in exchange for $90 of U.S. currency.

4. **SALARY**
Henry started a new job in which he is paid $9.50 an hour. Write and solve an equation to determine Henry's gross salary for a 40-hour work week.

5. **SALES TAX**
Amelia received a gift card to a local music shop for her birthday. She plans to use the gift card to buy some new CDs.

   a. Amelia chose 3 CDs that each cost $16. The sales tax on the three CDs is $3.96. Write a direct variation equation relating sales tax to the price.

   b. Graph the equation you wrote in part a.

   c. What is the sales tax rate that Amelia is paying on the CDs?
Arithmetic Sequences as Linear Functions

### Example 1
Determine whether the sequence 1, 3, 5, 7, 9, 11, . . . is an arithmetic sequence. Justify your answer.

If possible, find the common difference between the terms. Since $3 - 1 = 2$, $5 - 3 = 2$, and so on, the common difference is 2.

Since the difference between the terms of 1, 3, 5, 7, 9, 11, . . . is constant, this is an arithmetic sequence.

### Example 2
Write an equation for the $n$th term of the sequence 12, 15, 18, 21, . . .

In this sequence, $a_1$ is 12. Find the common difference.

$$d = 15 - 12 = 3$$

Use the formula for the $n$th term to write an equation.

$$a_n = a_1 + (n - 1)d$$

Formulate for the $n$th term

$$a_n = a_1 + (n - 1)d$$

The common difference is 3.

Simplify.

The equation for the $n$th term is $a_n = 3n + 9$.

### Exercises

Determine whether each sequence is an arithmetic sequence. Write yes or no. Explain.

1. 1, 1, 5, 9, 13, 17, . . .
   - yes; $d = 4$
2. 2, 5, 8, 11, 14, . . .
   - yes; $d = 3$
3. 1, 3, 5, 7, 9, 11, . . .
   - no; no common difference
4. 4, 9, 13, 17, 21, 25, . . .
5. 5, 10, 15, 20, 25, . . .
6. 29, 35, 41, 47, . . .
7. 29, 33, 37, . . .

Find the next three terms of each arithmetic sequence.

1. 4, 9, 13, 17, 21, 25, 29
2. 5, 10, 15, 20, 25, 29
3. 29, 33, 37, 41, 45, 49

Write an equation for the $n$th term of each arithmetic sequence. Then graph the first five terms of the sequence.

1. $a_n = 2n - 1$
2. $a_n = -3n + 2$
3. $a_n = -5n + 1$

b. Graph the function.

2. Refreshments You agree to pour water into the cups for the Booster Club at a football game. The pitcher contains 64 ounces of water when you begin. After you have filled 8 cups, the pitcher is empty and must be refilled.

   a. Write a function to represent the arithmetic sequence.
   - $a_n = -8n$

b. Graph the function.
**3-5 Skills Practice**

**Arithmetic Sequences as Linear Functions**

Determine whether each sequence is an arithmetic sequence. Write yes or no. Explain.

1. 4, 7, 9, 12, . . . no
2. 15, 13, 11, 9, . . . yes; −2
3. 7, 10, 13, 16, . . . yes; 3
4. −6, −5, −3, −1, . . . no
5. −5, −3, −1, 1, . . . yes; 2
6. −9, −12, −15, −18, . . . yes; −3
7. 10, 15, 25, 40, . . . no
8. −10, −5, 0, 5, . . . yes; 5

Find the next three terms of each arithmetic sequence.

9. 3, 7, 11, 15, . . . 19, 23, 27
10. 10, 20, 40, 80, . . .
11. −13, −11, −9, −7, . . . −5, −3, −1
12. −2, −5, −8, −11, . . . −14, −17, −20
13. 19, 24, 29, 34, . . . 39, 44, 49
14. 16, 7, −2, −11, . . . −20, −29, −38
15. 25, 7, 5, 10, . . . 12.5, 15, 17.5
16. 3, 4, 5, 6, . . . 7, 8, 9, 11

Write an equation for the nth term of each arithmetic sequence. Then graph the first five terms of the sequence.

17. 7, 13, 19, 25, . . . \(a_n = 6n + 1\)
18. 30, 26, 22, 18, . . . \(a_n = -4n + 34\)
19. −7, −4, −1, 2, . . . \(a_n = 3n - 10\)

20. **VIDEO DOWNLOADING** Brian is downloading episodes of his favorite TV show to play on his personal media device. The cost to download 1 episode is $1.99. The cost to download 2 episodes is $3.98. The cost to download 3 episodes is $5.97. Write a function to represent the arithmetic sequence.

\(a_n = 1.99n\)

**3-5 Practice**

**Arithmetic Sequences as Linear Functions**

Determine whether each sequence is an arithmetic sequence. Write yes or no. Explain.

1. 21, 13, 5, . . . yes; \(d = -8\)
2. −5, −12, 29, 46, . . . yes; \(d = 17\)
3. −2.2, −1.1, 0.1, 1.3, . . . no; no common difference
4. 1, 4, 9, 16, . . . no; no common difference
5. 5, −3, −1, 1, . . . yes; \(d = 2\)
6. 1, 4, 9, 16, . . . no; no common difference

Find the next three terms of each arithmetic sequence.

7. 8, 12, 16, . . . 20, 24, 28
8. −49, −35, −21, −7, . . . −9, −3, 3
9. 58, 52, 46, 40, . . . 34, 28, 22
10. 18, 20, 22, 24, . . . 30, 32, 34
11. 3, 5, 7, 9, . . . 11, 13, 15
12. 1, 2, 3, 4, . . . 5, 6, 7

Write an equation for the nth term of each arithmetic sequence. Then graph the first five terms of the sequence.

13. 3, 6, 9, 12, . . . \(a_n = 3n\)
14. 1, 4, 7, 10, . . . \(a_n = 3n - 2\)
15. 2, 5, 8, 11, . . . \(a_n = 3n - 3\)
16. 4, 7, 10, 13, . . . \(a_n = 3n - 1\)

16. **BANKING** Chem deposited $115.00 in a savings account. Each week thereafter, he deposits $35.00 into the account.

a. Write a function to represent the total amount Chem has deposited for any particular number of weeks after his initial deposit.

\(a_n = 35n + 115\)

b. How much has Chem deposited 30 weeks after his initial deposit?

$1165

17. **STORE DISPLAYS** Tamika is stacking boxes of tissue for a store display. Each row of tissues has 2 fewer boxes than the row below. The first row has 23 boxes of tissues.

a. Write a function to represent the arithmetic sequence.

\(a_n = -2n + 25\)

b. How many boxes will there be in the tenth row? 5
4. **NUMBER THEORY** One of the most famous sequences in mathematics is the Fibonacci sequence. It is named after Leonardo de Pisa (1170–1250) or Filius Bonacci, alias Leonardo Fibonacci. The first several numbers in the Fibonacci sequence are: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, . . . Does this represent an arithmetic sequence? Why or why not?

5. **SAVINGS** Inga's grandfather decides to contribute to her college education. He makes an initial contribution of $3000 and each month deposits an additional $500. After one month he will have contributed $3500.

   a. Write an equation for the nth term of the sequence. \( a_n = 3000 + 500n \)

   b. How much money will Inga's grandfather have contributed after 24 months? $15,000

   c. How much more money will Inga's grandfather have contributed after 25 rows? 68 seats

   d. Find the sum of each arithmetic series.

   1. \( 3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 \) 108
   2. \( 10 + 15 + 20 + 25 + 30 + 35 + 40 + 45 + 50 \) 270
   3. \( -21 + (-16) + (-11) + (-6) + (-1) + 4 + 9 + 14 \) -28
   4. even whole numbers from 2 through 100 2550
   5. odd whole numbers between 0 and 100 2500
Proportional Relationships

If the relationship between the domain and range of a relation is linear, the relationship can be described by a linear equation. If the equation passes through \((0, 0)\) and is of the form \(y = kx\), then the relationship is proportional.

**Example**

**COMPACT DISCS**

Suppose you purchased a number of packages of blank compact discs. If each package contains 3 compact discs, you could make a chart to show the relationship between the number of packages of compact discs and the number of discs purchased. Use \(x\) for the number of packages and \(y\) for the number of compact discs.

Make a table of ordered pairs for several points of the graph.

<table>
<thead>
<tr>
<th>Number of Packages</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of CDs</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
</tr>
</tbody>
</table>

The difference in the \(x\) values is 1, and the difference in the \(y\) values is 3. This pattern shows that \(y\) is always 3 times \(x\). This suggests the relation \(y = 3x\). Since the relation is also a function, we can write the equation in function notation as \(f(x) = 3x\).

The relation includes the point \((0, 0)\) because if you buy 0 packages of compact disks, you will not have any compact discs. Therefore, the relationship is proportional.

**Exercises**

1. **NATURAL GAS**

   Natural gas use is often measured in "therms." The total amount a gas company will charge for natural gas use is based on how much natural gas a household uses. The table shows the relationship between natural gas use and the total cost.

<table>
<thead>
<tr>
<th>Gas Used (therms)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost ($)</td>
<td>1.30</td>
<td>2.60</td>
<td>3.90</td>
<td>5.20</td>
</tr>
</tbody>
</table>

   a. Graph the data. What can you deduce from the pattern about the relationship between the number of therms used and the total cost?
   
   The relationship is proportional.
   
   b. Write an equation to describe this relationship. \(y = 1.30x\)
   
   c. Use this equation to predict how much it will cost if a household uses 40 therms.
   
   \(52.00\)

   **Example**

   Write an equation in functional notation for the relation shown in the graph.

   Select points from the graph and place them in a table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>-2</td>
<td>-4</td>
</tr>
</tbody>
</table>

   The difference between the \(x\)-values is 1, while the difference between the \(y\)-values is -2. This suggests that \(y = -2x\).

   If \(x = 1\), then \(y = -2(1) = -2\). But the \(y\)-value for \(x = 1\) is 0.

   \(y\) is always 2 more than \(-2x\).

   This pattern shows that 2 should be added to one side of the equation. Thus, the equation is \(y = -2x + 2\).

   **Exercises**

   Write an equation in function notation for the relation shown in the table. Then complete the table.

   1. \(f(x) = 4x + 2\)
   2. \(f(x) = -3x + 4\)

   Write an equation in function notation for each relation.

   3. \(f(x) = -x + 2\)
   4. \(f(x) = 2x + 2\)
Skills Practice

Proportional and Nonproportional Relationships

Write an equation in function notation for each relation.

1. \( f(x) = -2x \)

2. \( f(x) = x - 2 \)

3. \( f(x) = 1 - x \)

4. \( f(x) = x + 6 \)

5. \( f(x) = 5 - x \)

6. \( f(x) = 2x - 1 \)

7. GAMESHOWS The table shows how many points are awarded for answering consecutive questions on a game show.

<table>
<thead>
<tr>
<th>Question answered</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points awarded</td>
<td>200</td>
<td>400</td>
<td>600</td>
<td>800</td>
<td>1000</td>
</tr>
</tbody>
</table>

a. Write an equation for the data given. \( y = 200x \)

b. Find the number of points awarded if 9 questions were answered. 1800

Practice

Proportional and Nonproportional Relationships

1. BIOLOGY Male fireflies flash in various patterns to signal location and perhaps to ward off predators. Different species of fireflies have different flash characteristics, such as the intensity of the flash, its rate, and its shape. The table below shows the rate at which a male firefly is flashing.

<table>
<thead>
<tr>
<th>Times (seconds)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Flashes</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

a. Write an equation in function notation for the relation. \( f(t) = 2t \), where \( t \) is the time in seconds and \( f(t) \) is the number of flashes.

b. How many times will the firefly flash in 20 seconds? 40

2. GEOMETRY The table shows the number of diagonals that can be drawn from one vertex in a polygon. Write an equation in function notation for the relation and find the number of diagonals that can be drawn from one vertex in a 12-sided polygon. \( f(s) = s - 3 \), where \( s \) is the number of sides and \( f(s) \) is the number of diagonals.

Write an equation in function notation for each relation.

3. \( f(x) = -\frac{1}{2}x \)

4. \( f(x) = 3x - 6 \)

5. \( f(x) = 2x + 4 \)

For each arithmetic sequence, determine the related function. Then determine if the function is proportional or nonproportional. Explain.

6. 1, 3, 5, . . .

7. 2, 7, 12, . . .

8. -3, -6, -9, . . .

- \( a(n) = 2n - 1 \); nonproportional
- \( a(n) = 5n - 3 \); nonproportional
- \( a(n) = -3n \); proportional

not of form \( y = kx \)
not of form \( y = kx \)
of form \( y = kx \)
Word Problem Practice
Proportional and Nonproportional Relationships

1. ONLINE SHOPPING Ricardo is buying computer cables from an online store. If he buys 4 cables, the total cost will be $24. If he buys 5 cables, the total cost will be $29. If the total cost can be represented by a linear function, will the function be proportional or nonproportional? Explain.

2. FOOD It takes about four pounds of grapes to produce one pound of raisins. The graph shows the relation for the number of pounds of grapes needed, x, to make y pounds of raisins. Write an equation in function notation for the relation shown. f(x) = 0.25x

3. PARKING Palmer Township recently installed parking meters in their municipal lot. The cost to park for h hours is represented by the equation C = 0.25h.

   a. Make a table of values that represents this relationship.
   b. Describe the relationship between the time parked and the cost.

4. MUSIC A measure of music contains the same number of beats throughout the song. The table shows the relation for the number of beats counted after a certain number of measures have been played in the six-eight time. Write an equation to describe this relationship.

<table>
<thead>
<tr>
<th>Measures Played (m)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Number of Beats (b)</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
</tr>
</tbody>
</table>

5. GEOMETRY A fractal is a pattern containing parts which are identical to the overall pattern. The following geometric pattern is a fractal.

   a. Complete the table.

<table>
<thead>
<tr>
<th>Term</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Smaller Triangles</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
</tr>
</tbody>
</table>

   b. What are the next three numbers in the pattern? 25, 36, 49

   c. Write an equation in function notation for the pattern. f(x) = x^2

6. Describe the form of equations that have the same graph in both the usual coordinate plane and the taxicab plane.

   a. Taxicab Graphs

   You have used a rectangular coordinate system to graph equations such as y = x - 1 on a coordinate plane. In a coordinate plane, the numbers in an ordered pair (x, y) can be any two real numbers.

   A taxicab plane is different from the usual coordinate plane. The only points allowed are those that exist along the horizontal and vertical grid lines. You may think of the points as taxis that must stay on the streets.

   The taxicab graph shows the equations y = -2 and y = x - 1. Notice that one of the graphs is no longer a straight line. It is now a collection of separate points.

   Graph these equations on the taxicab plane at the right.

   1. y = x + 1
   2. y = -2x + 3
   3. y = 2.5
   4. x = -4

   Use your graphs for these problems.

   5. Which of the equations has the same graph in both the usual coordinate plane and the taxicab plane? x = -4

   6. Describe the form of equations that have the same graph in both the usual coordinate plane and the taxicab plane: x = A and y = B, where A and B are integers

   In the taxicab plane, distances are not measured diagonally, but along the streets. Write the taxi-distance between each pair of points.

   7. (0, 0) and (5, 2) 8. (0, 0) and (-3, 2) 9. (0, 0) and (2, 1.5)
   7 units 5 units 3.5 units

   10. (1, 2) and (4, 3) 11. (2, 4) and (-1, 3) 12. (0, 4) and (-2, 0)
   4 units 4 units 6 units

   Draw these graphs on the taxicab grid at the right.

   13. The set of points whose taxi-distance from (0, 0) is 2 units. indicated by X

   14. The set of points whose taxi-distance from (2, 1) is 3 units. indicated by dots
ERROR: undefined
OFFENDING COMMAND:

STACK: