Chapter 11

NAME ______________________ DATE __________ PERIOD __________

11-1 Study Guide and Intervention

Inverse Variation

Identify and Use Inverse Variations

An inverse variation is an equation in the form of \( y = \frac{k}{x} \) or \( xy = k \). If two points \((x_1, y_1)\) and \((x_2, y_2)\) are solutions of an inverse variation, then \( x_1 \cdot y_1 = k \) and \( x_2 \cdot y_2 = k \). From the product rule, you can form the proportion \( \frac{x_1}{x_2} = \frac{y_2}{y_1} \).

Example

If \( y \) varies inversely as \( x \) and \( y = 12 \) when \( x = 4 \), find \( x \) when \( y = 18 \).

Method 1 Use the product rule.

\[
\begin{align*}
4 \cdot 12 &= x_1 \cdot y_1 \\
48 &= x_1 \cdot 12 \\
4 &= x_1 \\
8 &= y_1 \\
5 &= \frac{8}{3}
\end{align*}
\]

Simplify.

Method 2 Use a proportion.

\[
\frac{x_1}{x_2} = \frac{y_2}{y_1}
\]

\[
\frac{4}{x_2} = \frac{18}{12}
\]

Cross multiply.

\[
48 = 18x_2
\]

Divide each side by 18.

\[
\frac{4}{x_2} = \frac{18}{12}
\]

Both methods show that \( x_2 = \frac{8}{3} \) when \( y = 18 \).

Exercises

Determine whether each table or equation represents an inverse or a direct variation. Explain.

1. \( x \quad y \quad \) direct variation; of the form \( y = kx \)
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   3 & 4 \\
   5 & 10 \\
   8 & 16 \\
   12 & 24 \\
   \end{array}
   \]

2. \( x \quad y \quad \) direct variation; of the form \( y = kx \)
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   3 & 4 \\
   5 & 10 \\
   8 & 16 \\
   12 & 24 \\
   \end{array}
   \]

3. \( xy = 15 \) inverse variation; of the form \( xy = k \)
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   3 & 4 \\
   5 & 8 \\
   6 & 10 \\
   10 & 3 \\
   \end{array}
   \]

Assume that \( y \) varies inversely as \( x \). Write an inverse variation equation that relates \( x \) and \( y \). Then solve.

4. If \( y = 10 \) when \( x = 5 \), find \( y \) when \( x = 2 \).
   \[
   x = 2, \; y = 50; \; 25
   \]

5. If \( y = 8 \) when \( x = -2 \), find \( y \) when \( x = 4 \).
   \[
   x = 4, \; y = -16; \; -4
   \]

6. If \( y = 100 \) when \( n = 120 \), find \( x \) when \( y = 20 \).
   \[
   x = 12,000; \; 600
   \]

7. If \( y = -16 \) when \( x = 4 \), find \( x \) when \( y = 32 \).
   \[
   x = -64; \; -2
   \]

8. If \( y = -7.5 \) when \( x = 25 \), find \( y \) when \( x = 5 \).
   \[
   x = 5, \; y = -187.5; \; -37.5
   \]

9. DRIVING The Gerardi family can travel to Oshkosh, Wisconsin, from Chicago, Illinois, in 4 hours if they drive an average of 45 miles per hour. How long would it take them if they increased their average speed to 50 miles per hour? \( 3.6 \text{ h} \)

10. GEOMETRY For a rectangle with given area, the width of the rectangle varies inversely as the length. If the width of the rectangle is 40 meters when the length is 5 meters, find the width of the rectangle when the length is 20 meters. \( 10 \text{ m} \)
Graph Inverse Variations

Inverse Variation Equation

An equation of the form \( xy = k \), where \( k \neq 0 \)

Example 1

Suppose you drive 200 miles without stopping. The time it takes to travel a distance varies inversely as the rate at which you travel. Let \( x \) = speed in miles per hour and \( y \) = time in hours. Graph the variation.

The equation \( xy = 200 \) can be used to represent the situation. Use various speeds to make a table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>30</td>
<td>6.7</td>
</tr>
<tr>
<td>40</td>
<td>5</td>
</tr>
<tr>
<td>50</td>
<td>4</td>
</tr>
<tr>
<td>60</td>
<td>3.3</td>
</tr>
</tbody>
</table>

Example 2

Graph an inverse variation in which \( y \) varies inversely as \( x \) and \( y = 3 \) when \( x = 12 \).

Solve for \( k \).

\[
\begin{align*}
xy &= k \\
12(3) &= k \\
36 &= k
\end{align*}
\]

Choose values for \( x \) and \( y \), which have a product of 36.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>-6</td>
</tr>
<tr>
<td>-3</td>
<td>-12</td>
</tr>
<tr>
<td>-2</td>
<td>-18</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

Exercises

Graph each variation if \( y \) varies inversely as \( x \).

1. \( y = 9 \) when \( x = -3 \)
2. \( y = 12 \) when \( x = 4 \)
3. \( y = -25 \) when \( x = 5 \)
4. \( y = 4 \) when \( x = 5 \)
5. \( y = -18 \) when \( x = -9 \)
6. \( y = 4.8 \) when \( x = 5.4 \)
7. \( y = 10 \) when \( x = 1 \)
8. \( y = 2 \) when \( x = 5 \)
9. \( y = 3 \) when \( x = 3 \)
10. \( y = -6 \) when \( x = -2 \)
11. \( y = -24 \) when \( x = 3 \)
12. \( y = 5 \) when \( x = 1 \)
13. \( y = 48 \) when \( x = 4 \)
14. \( y = -4 \) when \( x = \frac{1}{2} \)
11-1 Practice

Inverse Variation

Determine whether each table or equation represents an inverse or a direct variation. Explain.

1. \( \frac{4}{x} = \frac{1}{y} ; \quad y = \frac{8}{x} \)

2. \( xy = 20 \)

3. \( xy = -6 \)

4. \( xy = 2 \)

Assume that \( y \) varies inversely as \( x \). Write an inverse variation equation that relates \( x \) and \( y \). Then graph the equation.

5. \( y = -2 \) when \( x = -12 \)

6. \( y = -5 \) when \( x = 2 \)

7. \( y = 2.5 \) when \( x = 2 \)

\( xy = 24 \)

\( xy = 30 \)

\( xy = 5 \)

Write an inverse variation equation that relates \( x \) and \( y \). Assume that \( y \) varies inversely as \( x \). Then solve.

8. \( y = 124 \) when \( x = 12 \), find \( y \) when \( x = -24 \).

9. \( y = -8.5 \) when \( x = 6 \), find \( y \) when \( x = -2.5 \).

10. \( y = 3.2 \) when \( x = -5.5 \), find \( y \) when \( x = -6.4 \).

11. \( y = 0.6 \) when \( x = 7.5 \), find \( y \) when \( x = -1.25 \).

12. **EMPLOYMENT** The manager of a lumber store schedules 6 employees to take inventory in an 8-hour work period. The manager assumes all employees work at the same rate.

   a. Suppose 2 employees call in sick. How many hours will 4 employees need to take inventory?

   b. If the district supervisor calls in and says she needs the inventory finished in 6 hours, how many employees should the manager assign to take inventory?

13. **TRAVEL** Jesse and Joaquin can drive to their grandparents' home in 3 hours if they average 50 miles per hour. Since the road between the homes is winding and mountainous, their parents prefer they average between 40 and 45 miles per hour. How long will it take to drive to the grandparents' home at the reduced speed?

   a. between 3 h 20 min and 3 h 45 min

**11-1 Word Problem Practice**

Inverse Variation

1. **PHYSICAL SCIENCE** The illumination \( I \) produced by a light source varies inversely as the square of the distance \( d \) from the source. The illumination produced 5 feet from the light source is 80 foot-candles.

   \( I = \frac{k}{d^2} \)

   \( 80 = \frac{k}{5^2} \)

   Find the illumination produced 8 feet from the same source.

2. **MONEY** A formula called the Rule of 72 approximates how fast money will double in a savings account. It is based on the relation that the number of years it takes for money to double varies inversely as the annual interest rate. Use the information in the table to write the Rule of 72 formula.

<table>
<thead>
<tr>
<th>Years to Double</th>
<th>Annual Interest Rate (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>4</td>
</tr>
<tr>
<td>14.4</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>10.29</td>
<td>7</td>
</tr>
</tbody>
</table>

   \( yr = 72 \)

3. **ELECTRICITY** The resistance, in ohms, of a certain length of electric wire varies inversely as the square of the diameter of the wire. If a wire 0.04 centimeter in diameter has a resistance of 0.09 ohm, what is the resistance of a wire of the same length and material that is 0.08 centimeters in diameter?

   \( 0.15 \text{ ohm} \)

4. **BUSINESS** In the manufacturing of a certain digital camera, the cost of producing the camera varies inversely as the number produced. If 15,000 cameras are produced, the cost is $80 per unit.

   \( k = 15000 \times 80 \)

   Graph the relationship and label the point that represents the cost per unit to produce 25,000 cameras.

5. **SOUND** The sound produced by a string inside a piano depends on its length. The frequency of a vibrating string varies inversely as its length.

   \( f \times \ell = k \)

   Suppose a piano string 2 feet long vibrates 300 cycles per second. What would be the frequency of a string 4 feet long?

   150 cycles per second
Chapter 11

11-1 Enrichment

Direct or Indirect Variation

 Fill in each table below. Then write inversely, or directly to complete each conclusion.

1. \( f \) 2 4 8 16 32
   \( W \) 4 4 4 4 4
   \( A \) 8 16 32 64 128

For a set of rectangles with a width of 4, the area varies \( \text{directly} \) as the length.

2. Hours 2 4 5 6
   Speed 55 55 55 55
   Distance 110 220 275 330

For a car traveling at 55 mi/h, the distance covered varies \( \text{directly} \) as the hours driven.

3. Oat Bran \( \frac{1}{3} \) cup \( \frac{2}{3} \) cup 1 cup
   Water 1 cup 2 cup 3 cup
   Servings 1 2 3

The number of servings of oat bran varies \( \text{directly} \) as the number of cups of oat bran.

4. Hours of Work 128 128 128
   People Working 2 4 8
   Servings 64 32 16

A job requires 128 hours of work. The number of hours each person works varies \( \text{inversely} \) as the number of people working.

5. Miles 100 100 100 100
   Rate 20 25 50 100
   Hours 5 4 2 1

For a 100-mile car trip, the time the trip takes varies \( \text{inversely} \) as the average rate of speed the car travels.

6. \( b \) 1 4 5 6
   \( h \) 10 10 10 10
   \( A \) 50 20 25 30

For a set of right triangles with a height of 10, the area varies \( \text{directly} \) as the base.

Use the table at the right.

7. \( x \) varies \( \text{directly} \) as \( y \).
   \( x \) 1 1.5 2 2.5 3
   \( y \) 2 3 4 5 6
   \( x \) 60 40 30 24 20

8. \( z \) varies \( \text{inversely} \) as \( y \).

9. \( z \) varies \( \text{inversely} \) as \( z \).

Chapter 11

10 Glencoe Algebra 1

11-2 Study Guide and Intervention

Rational Functions

Identify Excluded Values The function \( y = \frac{x}{x-3} \) is an example of a rational function.
Because division by zero is undefined, any value of a variable that results in a denominator of zero must be excluded from the domain of that variable. These are called \( \text{excluded values} \) of the rational function.

Example

Fill in each table below. Then write inversely, or directly to complete each conclusion.

1. \( x \) varies \( \text{directly} \) as \( y \).
   \( x \) 1 1.5 2 2.5 3
   \( y \) 2 3 4 5 6
   \( x \) 60 40 30 24 20

2. Hours 2 4 5 6
   Speed 55 55 55 55
   Distance 110 220 275 330

For a car traveling at 55 mi/h, the distance covered varies \( \text{directly} \) as the hours driven.

3. Oat Bran \( \frac{1}{3} \) cup \( \frac{2}{3} \) cup 1 cup
   Water 1 cup 2 cup 3 cup
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   People Working 2 4 8
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A job requires 128 hours of work. The number of hours each person works varies \( \text{inversely} \) as the number of people working.

5. Miles 100 100 100 100
   Rate 20 25 50 100
   Hours 5 4 2 1

For a 100-mile car trip, the time the trip takes varies \( \text{inversely} \) as the average rate of speed the car travels.

Use the table at the right.

7. \( x \) varies \( \text{directly} \) as \( y \).
   \( x \) 1 1.5 2 2.5 3
   \( y \) 2 3 4 5 6
   \( x \) 60 40 30 24 20

8. \( z \) varies \( \text{inversely} \) as \( y \).

9. \( z \) varies \( \text{inversely} \) as \( z \).

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### Rational Functions

**Identify and Use Asymptotes**

Because excluded values are undefined, they affect the graph of the function. An asymptote is a line that the graph of a function approaches. A rational function in the form \( y = \frac{a}{x - b} + c \) has a vertical asymptote at the \( x \)-value that makes the denominator equal zero, \( x = b \). It has a horizontal asymptote at \( y = c \).

**Example**

Identify the asymptotes of \( y = \frac{1}{x - 1} + 2 \). Then graph the function.

**Step 1** Identify and graph the asymptotes using dashed lines.
- Vertical asymptote: \( x = 1 \)
- Horizontal asymptote: \( y = 2 \)

**Step 2** Make a table of values and plot the points. Then connect them.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>3.5</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
</tr>
</tbody>
</table>

**Exercises**

Identify the asymptotes of each function. Then graph the function.

1. \( y = \frac{6}{x} \) \( x = 0 \)
2. \( y = \frac{2}{x - 2} \) \( x = 2 \)
3. \( y = \frac{x}{x + 6} \) \( x = -6 \)
4. \( y = \frac{x - 3}{x + 4} \) \( x = -4 \)
5. \( y = \frac{3x - 5}{x + 8} \) \( x = -8 \)
6. \( y = \frac{-5}{2x - 14} \) \( x = 7 \)
7. \( y = \frac{x}{3x + 21} \) \( x = -7 \)
8. \( y = \frac{x - 1}{3x - 36} \) \( x = 4 \)
9. \( y = \frac{9}{5x + 40} \) \( x = -8 \)
10. \( y = \frac{1}{x} \) \( x = 0, y = 0 \)
11. \( y = \frac{3}{x} \) \( x = 0, y = 0 \)
12. \( y = \frac{2}{x + 1} \) \( x = -1, y = 0 \)
13. \( y = \frac{3}{x - 2} \) \( x = 2, y = 0 \)
14. \( y = \frac{2}{x + 1} - 1 \) \( x = -1, y = -1 \)
15. \( y = \frac{1}{x + 3} \) \( x = 2, y = 3 \)
**11-2 Practice**

**Rational Functions**

State the excluded value for each function.

1. \( y = -\frac{3}{2} x + 5 \) : excluded value is 0
2. \( y = -\frac{3}{2} x - 5 \) : excluded value is 0
3. \( y = -\frac{3}{2} x + 5 \) : excluded value is 0
4. \( y = -\frac{x - 1}{2x + 3} x = -3 \)
5. \( y = \frac{x + 1}{2x + 3} x = -\frac{3}{2} \)
6. \( y = \frac{1}{2x - 2} x = \frac{2}{3} \)

Identify the asymptotes of each function. Then graph the function.

7. \( y = \frac{2}{x} x = 0, y = 0 \)
8. \( y = \frac{3}{x} x = 0, y = 0 \)
9. \( y = \frac{2}{x - 1} x = 1, y = 0 \)
10. \( y = \frac{2}{x + 2} x = -2, y = 0 \)
11. \( y = \frac{1}{x - 3} + 2 x = 3, y = 2 \)
12. \( y = \frac{2}{x + 1} - 1 x = -1, y = -1 \)

**Air Travel:** Denver, Colorado, is located approximately 1000 miles from Indianapolis, Indiana. The average speed of a plane traveling between the two cities is given by \( y = \frac{1000}{x} \), where \( x \) is the total flight time. Graph the function.

**Word Problem Practice**

**Rational Functions**

1. **Bullet Trains** The Shinkansen, or Japanese bullet train network, provides high-speed ground transportation throughout Japan. Trains regularly operate at speeds in excess of 200 kilometers per hour. The average speed of a bullet train traveling between Tokyo and Kyoto is given by \( y = \frac{515}{0.001} \), where \( x \) is the total travel time in hours. Graph the function.

2. **Driving** Peter is driving to his grandparents’ house 40 miles away. During the trip, Peter makes a 30-minute stop for lunch. The average speed of Peter’s trip is given by \( y = \frac{40}{x - 0.5} \), where \( x \) is the total time spent in the car. What are the asymptotes of the function? Explain why \( x = 0 \) cannot be an asymptote.

3. **Error Analysis** Nicolas is graphing the equation \( y = \frac{20}{x + 2} - 6 \) and draws a graph with asymptotes at \( y = 3 \) and \( x = -6 \). Explain the error that Nicolas made in his graph. The asymptotes should be \( x = 3, y = -6 \).

4. **Used Cars** While researching cars to purchase online, Ms. Jacobs found that the value of a used car is inversely proportional to the age of the car. The average price of a used car is given by \( y = \frac{17,900}{x + 1.2} + 100 \), where \( x \) is the age of the car. What are the asymptotes of the function? Explain why \( x = 0 \) cannot be an asymptote. 
\( x = -1.2, y = 100 \); An asymptote at \( x = 0 \) would give \( y \), the cost of the car, an infinite value when the car is brand new. The asymptote needs to be located at \( x = a \), where \( a < 0 \).

5. **Family Reunion** The Gaudet family is holding their annual reunion at Watkins Park. It costs $50 to get a permit to hold the reunion at the park, and the family is spending $8 per person on food. The Gaudets have agreed to split the cost of the event evenly among all those attending.

   a. Write an equation showing the cost per person if \( x \) people attend the reunion.
   \( y = 8 + \frac{50}{x} \)

   b. What are the asymptotes of the equation? \( x = 0, y = 8 \)

   c. Now assume that the family wants to let a long-lost cousin attend for free. Rewrite the equation to find the new cost per paying person. 
   \( y = 8 + \frac{58}{x - 1} \)

   d. What are the asymptotes for the new equation? \( x = 1, y = 8 \)
Example 1

Step 1 Plot points and draw a smooth solid curve. Because the inequality involves a greater than or equal to sign, solutions that satisfy \( y = \frac{1}{2} \) will be a part of the graph.

Step 2 Plot the asymptotes, \( x = 0 \) and \( y = 0 \), as dashed lines.

Step 3 Begin testing values. A value must be tested between each set of lines, including asymptotes.

Region 1 Test \((-1, 1)\). This returns a true value for the inequality.

Region 2 Test \((-1, -0.5)\). This returns a true value for the inequality.

Region 3 Test \((-1, 2)\). This returns a true value for the inequality.

Region 4 Test \((1, 2)\). This returns a true value for the inequality.

Region 5 Test \((1, 0.5)\). This returns a false value for the inequality.

Step 4 Shade the regions where the inequality is true.

Exercises

Graph each inequality.

1. \( y \leq \frac{1}{x + 1} \)
2. \( x > \frac{3}{2} \)
3. \( y \leq \frac{1}{x + 1} - 1 \)

11-3 Study Guide and Intervention

Simplifying Rational Expressions

Identify Excluded Values

Because a rational expression involves division, the denominator cannot equal zero. Any value of the denominator that results in division by zero is called an excluded value of the denominator.

Example 1

State the excluded value of \( \frac{4m - 8}{m + 2} \).

Exclude the values for which \( m + 2 = 0 \). The denominator cannot equal 0.

\[ m + 2 = 0 \]

Subtract 2 from each side.

\[ m = -2 \]

Therefore, \( m \) cannot equal \(-2\).

Example 2

State the excluded values of \( \frac{x^2 + 1}{x^2 - 9} \).

Exclude the values for which \( x^2 - 9 = 0 \). The denominator cannot equal 0.

\[ x^2 - 9 = 0 \]

Factor.

\[ (x + 3)(x - 3) = 0 \]

Therefore, \( x \) cannot equal \(-3 \) or 3.

Exercises

State the excluded values for each rational expression.

1. \( \frac{2b}{b^2 - 8} \), \(-\sqrt{8}, -\sqrt{8}\)
2. \( \frac{12 - a}{32 + a} \), \(-32\)
3. \( \frac{x^2 - 2}{x^2 + 4} \), \(2, -2\)
4. \( \frac{m^2 - 4}{2m^2 - 8} \), \(2, -2\)
5. \( \frac{2x - 12}{x^2 - 4} \), \(-2, 2\)
6. \( \frac{2x + 18}{x^2 - 16} \), \(-4, 4\)
7. \( \frac{x^2 - 4}{x^2 + 4 + x} \), \(-2\)
8. \( \frac{a - 1}{x^3 + 5x + 6} \), \(1, -3\)
9. \( \frac{b^2 + 3}{b^2 + 4b + 3} \), \(-3, -1\)
10. \( \frac{m^2 - 1}{2m^2 - m - 1} \), \(1, -2\)
11. \( \frac{25 - n^2}{n^2 - 4n - 5} \), \(-1, 5\)
12. \( \frac{3x^2 + 5x + 1}{x^2 - 10x + 16} \), \(2, 8\)
13. \( \frac{n^2 + 2n - 3}{n^2 + 4n - 5} \), \(5, 1\)
14. \( \frac{x^2 - y - 2}{3y^2 - 12} \), \(-2, 2\)
15. \( \frac{k^2 + 3k - 3}{m^2 - 20k + 64} \), \(4, 16\)
16. \( \frac{x^3 + 4x + 4}{4x^3 + 11x - 3} \), \(-3, 1/4\)
**Simplifying Rational Expressions**

**Example 1**

Simplify \( \frac{5x^2}{24yz} \).

- \( \frac{5x^2}{24yz} = \frac{6(5x^2)}{6(24yz)} \), where the GCF of the numerator and the denominator is 6.
- \( = \frac{6(5x^2)}{6(24yz)} \)
- \( = \frac{5x^2}{24yz} \/
- \( \cdot 6z \).

**Example 2**

Simplify \( \frac{3x - 9}{x^2 - 5x + 6} \). State the excluded values of \( x \).

- \( \frac{3x - 9}{x^2 - 5x + 6} = \frac{3(x - 3)}{(x - 2)(x - 3)} \)
- \( = \frac{3}{x - 2} \)
- Simplify.

Excluded values for which \( x^2 - 5x + 6 = 0 \):

\( (x - 2)(x - 3) = 0 \)

Therefore, \( x \neq 2 \) and \( x 
eq 3 \).

**Exercises**

Simplify each expression. State the excluded values of the variables.

1. \( \frac{10h}{a} \neq 0; \frac{b}{2} \neq 0 \)
2. \( \frac{7x^2}{21x^2}; n \neq 0 \)
3. \( \frac{x + 2}{x^2}; x \neq -2 \) or \( x \neq 2 \)
4. \( \frac{m^2 - 4}{m^2 + 5m + 6}; m \neq 2 \) or \(-2 \)
5. \( \frac{2n}{n + 4}; n \neq -4 \) or \( n \neq 4 \)
6. \( \frac{x^2 + x^2}{x^2 + 1}; x \neq -1 \) or \( x \neq 1 \)
7. \( \frac{x + 2}{x^2 + 4}; x \neq -2 \)
8. \( \frac{k + 1}{k + 1}; k \neq -3 \) or \( k \neq 3 \)
9. \( \frac{k^2}{k^2 + 4k + 3}; n \neq 5 \) or \( n \neq -1 \)
10. \( \frac{m + n}{m + n}; n \neq 5 \) or \( n \neq 0 \)
11. \( \frac{m + 1}{n - 2}; n \neq -4 \) or \( n \neq 2 \)
12. \( \frac{y^2 - 2}{y^2 + 4}; y \neq -2 \) or \( y \neq 2 \)
13. \( \frac{y^2 - 8}{y^2 + 16}; y \neq -2 \) or \( y \neq 2 \)
1. \( \frac{4n}{m^2 - 25} \), \(-7, 5\)

2. \( \frac{p^2 - 16}{p^2 - 13p + 36} \), \(4, 9\)

3. \( \frac{a^2 - 3a - 15}{a^2 + 8a + 15} \), \(-5, -3\)

Simplify each expression. State the excluded values of the variables.

4. \( \frac{12x}{4x^2 - 1} \), \(0\)

5. \( \frac{6x^2}{3x^2 - 12x + 27} \), \(0, 0, 0\)

6. \( \frac{36a^2p^2 - 9k}{20ab^2 + 5p^2} \), \(0, 0, 0\)

7. \( \frac{5c^2d^2}{8 + c^4} \), \(0, -2, d: 0\)

8. \( \frac{p^2 - 8p + 12}{p - 2} \), \(p = 6, 2\)

9. \( \frac{m + 3}{m^2 - 9} \), \(m = -3, 3\)

10. \( \frac{2x - 14}{2x^2 - 9b + 14} \), \(x = 2, 7\)

11. \( \frac{y + 8}{y^2 - 4y + 4} \), \(y = 2\)

12. \( \frac{r^2 - 7r + 6}{r^2 - 3r + 2} \), \(r = 6, 1\)

13. \( \frac{t^2 - 81}{t^2 - 12t + 27} \), \(t = -3, 3\)

14. \( \frac{2x^2 + 18x + 36}{3x^2 - 3x - 36} \), \(x = -3, 4\)

15. \( \frac{2(x + 6)(x - 4)}{(x - 3)(x - 4)} \), \(-3, 4\)

16. \( \frac{2(x + 9)y + 4}{4y^2 - 4y - 16} \), \(y = 4\)

17. \( \frac{2y^2 - 3y + 1}{2y^2 - 3y + 2} \), \(2, 1\)

18. \( \frac{2y^2 - 3y + 1}{2y^2 - 3y + 2} \), \(2, 1\)

19. ENTERTAINMENT

Fairfield High spent \( d \) dollars for refreshments, decorations, and advertising for a dance. In addition, they hired a band for \$550.

a. Write an expression that represents the cost of the hand as a fraction of the total amount spent for the school dance.

b. If \( d \) is \$1650, what percent of the budget did the band account for? 25%

20. PHYSICAL SCIENCE

Mr. Kaminksi plans to dislodge a tree stump in his yard by using a 6-foot bar as a lever. He places the bar so that 0.5 foot extends from the fulcrum to the end of the bar under the tree stump. In the diagram, \( b \) represents the total length of the bar and \( r \) represents the portion of the bar beyond the fulcrum.

a. Write an equation that can be used to calculate the mechanical advantage. \( MA = \frac{b}{r} \)

b. What is the mechanical advantage? 11

c. If a force of 200 pounds is applied to the end of the lever, what is the force placed on the tree stump? \( 2200 \) lb

21. GRAPHING

Recall that the slope of a line is a ratio of the vertical change to the horizontal change in coordinates for two given points. Write a rational expression that represents the slope of the line containing the points \((p, r)\) and \((7, -3)\).

22. PACKAGING

In order to safely ship a new electronic device, the distribution manager at Data Products Company determines that the package must contain a certain amount of cushioning on each side of the device. The device is shaped like a cube with side length \( x \), and some sides need more cushioning than others because of the device’s design. The volume of a shipping container is represented by the expression \((x + 3)(x + 4)(x - 3)\). Find the polynomial that represents the area of the top of the box if the height of the box is \( x + 2 \).

23. PHYSICAL SCIENCE

Pressure is equal to the magnitude of a force divided by the area over which the force acts.

\( P = \frac{F}{A} \)

Gabe and Shelby each push open a door with one hand. In order to open, the door requires 20 pounds of force. The surface area of Gabe’s hand is 10 square inches, and the surface area of Shelby’s hand is 8 square inches. Whose hand feels the greater pressure?

Shelby’s: \( \frac{25}{20} \) lb/in\(^2\) (vs Gabe’s \( \frac{2}{20} \) lb/in\(^2\))

24. AUTOMOBILES

The force needed to keep a car from skidding out of a turn on a particular road is given by the formula below. What force is required to keep a 2000-pound car traveling at 50 miles per hour on a curve with radius of 750 feet on the road? What value of \( r \) is excluded?

\( f = \frac{0.0672w - \frac{7}{30}}{r} \)

25. SCHOOL CHOICE

During a recent school year, the ratio of public school students to private school students in the United States was approximately 7.6 to 1. The students attending public school outnumbered those attending private schools by \( 42,240,000 \).

a. Write a rational expression to express the ratio of public school students to private school students.

\( \frac{x + 42,240,000}{x} \)

b. How many students attended private school? \( 6,400,000 \)
Shannon's Juggling Theorem

Mathematicians look at various mathematical ways to represent juggling. One way they have found to represent juggling is Shannon's Juggling Theorem. Shannon's Juggling Theorem uses the rational equation

\[
\frac{f + d}{v + a} = \frac{b}{h}
\]

where \( f \) is the flight time, or how long a ball is in the air, \( d \) is the dwell time, or how long a ball is in a hand, \( v \) is the vacant time, or how long a hand is empty, \( a \) is the number of balls, and \( h \) is the number of hands (either 1 or 2 for a real-life situation, possibly more for a computer simulation).

So, given the values for \( f, d, v, \) and \( h \), it is possible to determine the number of balls being juggled. If the flight time is 9 seconds, the dwell time is 3 seconds, the vacant time is 1 second, and the number of hands is 2, how many balls are being juggled?

\[
\frac{f + d}{v + a} = \frac{b}{h}
\]


\[
9 + 3 \quad = \quad \frac{b}{2}
\]

\[
\frac{b}{2} = \frac{12}{2}
\]

\[
24 = 4b
\]

So, the number of balls being juggled is 6.

Given the following information, determine the number of balls being juggled.

1. Flight time = 6 seconds, vacant time = 1 second, dwell time = 1 second, number of hands = 2

7

2. Flight time = 13 seconds, vacant time = 1 second, dwell time = 5 seconds, number of hands = 3

3

3. Flight time = 4 seconds, vacant time = 1 second, dwell time = 1 second, number of hands = 2

5

4. Flight time = 16 seconds, vacant time = 1 second, dwell time = 2 seconds, number of hands = 2

12

5. Flight time = 18 seconds, vacant time = 3 seconds, dwell time = 2 seconds, number of hands = 1

4

### Exercises

Find each product.

1. \( \frac{6ab}{a^3} \cdot \frac{6a}{b^2} \)

2. \( \frac{m + 1}{3} \cdot \frac{4p}{3} \)

3. \( \frac{x + 2}{x - 4} \cdot \frac{x + 2}{x - 1} \)

4. \( \frac{m - 5}{m} \cdot \frac{16}{2} \)

5. \( \frac{2x - 8}{x - 4} \cdot \frac{2 + 4}{x - 4} \)

6. \( \frac{x - 8}{x + 16} \cdot \frac{x - 8}{x + 16} \cdot \frac{x - 8}{x + 16} \)

7. \( \frac{x^2 + 6x + 2}{x + 6} \cdot \frac{2x - 1}{x + 1} \)

8. \( \frac{a^2 - 25}{a + 2} \cdot \frac{a - 5}{a + 5} \)

9. \( \frac{a^3 + 27}{a^2 + 3a + 9} \cdot \frac{a^3 - 27}{a - 3} \)

10. \( \frac{m + 1}{m + 1} \cdot \frac{m + 1}{m + 1} \)

11. \( \frac{a^2 + 7a + 12}{a^2 + 2a - 8} \cdot \frac{a^2 + 3a - 10}{a + 3} \cdot \frac{a^2 - 2a - 8}{a - 2} \cdot \frac{a + 4}{a - 4} \)

12. \( \frac{3p - 3r}{10p} \cdot \frac{2p^2 + 1}{p + r} \)

13. \( \frac{v - 7}{3v + 6} \cdot \frac{v^2 + 11v + 24}{v - 2} \)

14. \( \frac{v - 7}{3v + 6} \cdot \frac{v^2 + 8v}{v - 2} \cdot \frac{v - 7}{3v + 6} \)
### 11-4 Study Guide and Intervention (continued)

Multiplying and Dividing Rational Expressions

**Divide Rational Expressions** To divide rational expressions, multiply by the reciprocal of the divisor. Then simplify.

#### Example 1
Find \( \frac{12x^2}{5ab} \div \frac{10ab}{c^2} \).

\[
\frac{12x^2}{5ab} \div \frac{10ab}{c^2} = \frac{12x^2}{5ab} \cdot \frac{c^2}{10ab} = \frac{12x^2c^2}{50abc} = \frac{6x^2c^2}{25abc}
\]

#### Example 2
Find \( \frac{x^4 + 6x - 27}{x^2 + 11x + 18} \div \frac{x - 3}{x^3 + x - 2} \).

\[
\frac{x^4 + 6x - 27}{x^2 + 11x + 18} \div \frac{x - 3}{x^3 + x - 2} = \frac{x^4 + 6x - 27}{x^2 + 11x + 18} \cdot \frac{x^3 + x - 2}{x - 3} = \frac{(x + 9)(x - 3)}{(x + 9)(x + 2)} \cdot \frac{(x - 3)(x - 1)}{(x - 3)} = x - 1
\]

### Exercises

Find each quotient.

1. \( \frac{12x}{a^2} \div \frac{b}{p} \)
2. \( \frac{n}{4} \div \frac{p}{4} \)
3. \( \frac{3x^7}{8} \div 6xy \)
4. \( \frac{m}{8} \div \frac{n}{16} \)
5. \( \frac{2n - 4}{2n} \div \frac{3}{a + 2} \)
6. \( \frac{y^2 - 36}{y^2 - 9} \div \frac{y - 6}{y - 7} \)
7. \( \frac{a + 5b}{a^2} \div \frac{a^3}{a + 2} \)
8. \( \frac{9x + 5}{a - 2} \div \frac{a - 3}{2a + 1} \)
9. \( \frac{x^4 - 81}{x^2 - 9} \div \frac{x + 3}{x - 4} \)
10. \( \frac{x^2 - 4}{x - 2} \div \frac{x^2 - 4}{x + 2} \)
11. \( \frac{a^2 - 9}{a^2 - 3a} \div \frac{(a - 3)(a - 3)}{(a - 2)(a - 3)} \)
12. \( \frac{a^2 - 4}{a^2 + 3a + 2} \div \frac{a^2 - 16}{a^2 - 16} \)
13. \( \frac{a^2 + 2a + 1}{a^2 + 3a + 2} \div \frac{(a + 1)(a + 1)}{a + 1} \)
14. \( \frac{b^2 - 1}{b^2 - 4} \div \frac{b^2 - 2b}{b^2 - 2b} \)
15. \( \frac{x^2 - 25}{x^2 - 10x + 25} \div \frac{x^2 - 10x + 25}{x^2 - 10x + 25} \)
16. \( \frac{x^2 + a^2}{x^2 - 1} \div \frac{(a - 1)(a + 1)}{a + 1} \)
17. \( \frac{a^2 - 4a + 4}{a^2 - 2a + 1} \div \frac{(a - 2)(a - 2)}{(a - 2)(a - 2)} \)
18. \( \frac{x^2 - 9}{x^2 - 6} \div \frac{x^2 - 9}{x^2 - 6} \)
19. \( \frac{a^2 - 2ab + 1}{a^2 - 2ab + 1} \div \frac{(a - 1)(a - 1)}{a - 1} \)
20. \( \frac{x^2 + 10x + 25}{x^2 - 9} \div \frac{5y + 5}{y - 3} \)
21. \( \frac{b - 4}{b^2 - 16} \div \frac{b - 4}{b - 4} \)

Answers

### 11-4 Skills Practice

Multiplying and Dividing Rational Expressions

Find each product.

1. \( \frac{14}{c} \div \frac{7c^2}{c^2} \)
2. \( \frac{2m}{3} \div \frac{12}{m^2} \)
3. \( \frac{2a}{b} \div \frac{2a}{c} \)
4. \( \frac{2x}{y} \div \frac{3y}{4} \)
5. \( \frac{3(m - 6)}{2m - 6} \cdot \frac{a - 2}{a + 2} \)
6. \( \frac{4a + 2}{4a - 2} \cdot \frac{a - 2}{a + 2} \)
7. \( \frac{(y - 3)(3 + y)}{4} \cdot \frac{8}{y + 3} \)
8. \( \frac{(x - 2)(x + 2)}{x(x + 2)} \cdot \frac{2(x + 2)}{x} \)
9. \( \frac{a - 7}{a + 7} \div \frac{a + 5}{a + 7} \)
10. \( \frac{b + 4}{b - 4} \div \frac{b - 4}{b + 4} \)

Find each quotient.

11. \( \frac{c^2}{d^2} \div \frac{c^2}{d^2} \)
12. \( \frac{c^2}{d^2} \div \frac{c^2}{d^2} \)
13. \( \frac{6x^3}{4x^2} \div \frac{12x}{9} \)
14. \( \frac{4m^3}{5m^2} \div \frac{2m^2}{5m^2} \)
15. \( \frac{3b + 3}{b + 2} \div \frac{(b + 1)}{3} \)
16. \( \frac{x - 5}{x + 3} \div \frac{(x - 5)(x + 3)}{x + 3} \)
17. \( \frac{x^2 - 12}{x} \div \frac{x + 3}{x} \)
18. \( \frac{x^2 - 9}{a + 6} \div \frac{a + 6}{a + 1} \)
19. \( \frac{m^2 + 2m + 1}{10m - 10} \div \frac{m + 1}{2(m + 1)} \)
20. \( \frac{x^2 + 10x + 25}{3y - 9} \div \frac{5y + 5}{y - 3} \)
21. \( \frac{b - 4}{b - 4} \div \frac{b - 4}{b - 4} \)
11-4 Practice

Multiplying and Dividing Rational Expressions

Find each product.

1. \( \frac{16x^3}{9y} \cdot \frac{10y^3}{9x^2} \)
2. \( \frac{24x^4}{9y^3} \cdot \frac{12y^3}{18x^2} \)
3. \( \frac{3x + 2a + 2b}{5x^2} \cdot \frac{7x - 2a}{3x + 2a} \)
4. \( \frac{m + 7}{m - 6(m + 2)} \cdot \frac{(m - 6m + 4)}{m + 2} \)
5. \( \frac{a - 4}{a + 12} \cdot \frac{a + 3}{a - 6} \)
6. \( \frac{4x + 8}{x + 2} \cdot \frac{x^2 - 5x + 4}{x - 7} \)
7. \( \frac{5a + 10b + 16}{5a - 10} \cdot \frac{a - 2}{a^2 + 9a + 8} \)
8. \( \frac{3y - 9}{y + 2} \cdot \frac{y^2 - 9y + 16}{y - 3} \)
9. \( \frac{b^2 + 6a - 4}{b^2 - 36} \cdot \frac{(b + 1)(b - 1)}{(b - 6)(b - 2)} \)
10. \( \frac{t^2 - 9t + 20}{t^2 - 10t + 25} \cdot \frac{t^2 + 7t + 12}{t - 5} \)

11. \( \frac{29x^3}{25y^2} \div \frac{289}{3xy} \)
12. \( \frac{20}{5y^2} \div \frac{36y^2}{5xy} \)
13. \( \frac{2y}{a + 1} \div \frac{2y}{a + 1} \)
14. \( \frac{z^2 - 4}{z - 2} \div \frac{3y}{z - 2} \)
15. \( \frac{6x + 12}{6x - 2} \div \frac{2x + 6}{x + 3} \)
16. \( \frac{5a - 10}{5a + 10} \div \frac{2a - 4a + 12}{5a - 10} \)
17. \( \frac{a + 6a}{a + 5} \div \frac{a + 5}{a - 6} \)
18. \( \frac{5x - 6}{5x + 9} \div \frac{5x - 6}{5x + 9} \)
19. \( \frac{3y^2 + 7y - 18}{3y^2 + 7y - 18} \div \frac{y - 2}{y + 2} \)

20. \( \frac{9y^2 + 8y - 9}{9y^2 + 8y - 9} \div \frac{y - 2}{y + 2} \)

21. BIOLOGY The heart of an average person pumps about 9000 liters of blood per day. How many quarts of blood does the heart pump per hour? (Hint: One quart is equal to 0.946 liter.) Round to the nearest whole number. 369 q/h

22. TRAFFIC On Saturday, it took Ms. Torres 24 minutes to drive 20 miles from her home to her office. During Friday’s rush hour, it took her 75 minutes to drive the same distance.

a. What was Ms. Torres’s average speed in miles per hour on Saturday? 50 mph
b. What was her average speed in miles per hour on Friday? 16 mph

11-4 Word Problem Practice

Multiplying and Dividing Rational Expressions

1. JOBS Rosa earned $26.25 for babysitting for 1.75 hours. At this rate, how much will she earn babysitting for 5 hours? $73.75

2. HOMEWORK Alejandro and Ander were working on the following homework problem.

Find \( \frac{a - 10}{n + 3} \cdot \frac{2n + 6}{n + 3} \)

Alejandro’s Solution
\( n = 10, 2n + 6 \)
\( n + 3 \cdot n + 3 \)
\( = 2n - 10 \)
\( = 2n - 20 \)

Ander’s Solution
\( n = 10, 2n + 6 \)
\( n + 3 \cdot n + 3 \)
\( = 2n - 10 \)
\( = 2n - 20 \)

Is either of them correct? Explain. Alejandro is correct because he multiplied the denominators. Ander treated the denominator like it was an addition problem.

3. GEOMETRY Suppose the rational expression \( \frac{5km^2}{3a} \) represents the area of a section in a tiled floor and \( \frac{5km}{3a} \) represents the section’s length. Write a rational expression to represent the section’s width. \( \frac{5km^2}{4b} \)

4. TRAVEL Helene travels 800 miles from Amarillo to Brownsville at an average speed of 60 miles per hour. She makes the return trip driving an average of 40 miles per hour. What is the average rate for the entire trip? (Hint: Recall that \( \frac{t = d / r} {48} \) mph

5. MANUFACTURING India works in a metal shop and needs to drill equally spaced holes along a strip of metal. The centers of the holes on the ends of the strip must be exactly 1 inch from each end. The remaining holes will be equally spaced.

a. If there are \( x \) equally spaced holes, write an expression for the number of equal spaces there are between holes. \( x - 1 \)

b. Write an expression for the distance between the end screws if the length is \( 4 \). \( x - 2 \)

c. Write a rational equation that represents the distance between the holes on a piece of metal that is \( 4 \) inches long and must have \( x \) equally spaced holes. \( \frac{4 - 2}{3} \)

d. How many holes will be drilled in a metal strip that is 6 feet long with a distance of 7 inches between the centers of each screw? 11
Chapter 11

11-4 Enrichment

Geometric Series
A geometric series is a sum of the terms in a geometric sequence. Each term of a geometric sequence is formed by multiplying the previous term by a constant term called the common ratio.

1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} — geometric sequence where the common ratio is \frac{1}{2}

The sum of a geometric series can be represented by the rational expression \( S_n = \frac{x_0(1 - r^n)}{1 - r} \), where \( x_0 \) is the first term of the series, \( r \) is the common ratio, and \( n \) is the number of terms.

In the example above, \( 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 1 \cdot \frac{1}{2}^{1-1} = \frac{15}{8} \)

You can check this by entering \( 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \) on a calculator. The result is the same.

Rewrite each sum as a rational expression and simplify.

1. \( 9 + 3 + \frac{1}{3} + \frac{1}{9} = \frac{121}{9} \)

2. \( 500 + 250 + 125 + 62 \frac{1}{2} = \frac{1875}{2} \)

3. \( 6 + 1 + \frac{1}{6} + \frac{1}{36} = \frac{259}{36} \)

4. \( 100 + 20 + 4 + \frac{4}{5} = \frac{624}{5} \)

5. \( 1000 + 100 + 10 + 1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} = \frac{1,111,111}{1000} \)

6. \( 55 + 5 + \frac{5}{11} + \frac{5}{121} = \frac{7320}{121} \)

11-4 Spreadsheet Activity

Revolutions per Minute

One of the characteristics that makes a spreadsheet powerful is the ability to recalculate values in formulas automatically. You can use this ability to investigate real-world situations.

Example 1
Use a spreadsheet to investigate the effect of doubling the diameter of a tire on the number of revolutions the tire makes at a given speed.

Use dimensional analysis to find the formula for the revolutions per minute of a tire with diameter of \( x \) inches traveling at \( y \) miles per hour.

\[
\frac{1 \text{ revolution}}{\text{minute}} = \frac{y}{\frac{15}{2} \pi x} \frac{\text{revolution}}{\text{rotation}} \frac{\text{rotation}}{\text{minute}}
\]

\[
= \frac{y}{\frac{15}{2} \pi x} \text{ revolutions per minute}
\]

Step 1
Use Column A of the spreadsheet for diameter of the tire in inches. Use Column B for the speed in miles per hour.

Step 2
Column C contains the formula for the number of rotations per minute. Notice that in Excel, \( \pi \) is entered as PI().

Step 3
Choose values for the diameter and speed and study the results shown in the spreadsheet. It appears that when the diameter is doubled, the number of revolutions per minute is halved.

Exercises

1. How is the number of revolutions affected if the speed of a wheel of a given diameter is doubled? RPM is cut in half.

2. Name two ways that you can double the RPM of a bicycle wheel. Double the speed or halve the diameter of the wheel, keeping the same speed.

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**11-5 Study Guide and Intervention**

**Dividing Polynomials**

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

**Example 1** Find $(4x^2 - 12r) ÷ (2r)$.

\[
(4x^2 - 12r) ÷ 2r = \frac{4x^2}{2r} - \frac{12r}{2r}
\]

\[
= \frac{2x}{r} - 6
\]

Simplify.

**Example 2** Find $(3x^2 - 8x + 4) ÷ (4x)$.

\[
(3x^2 - 8x + 4) ÷ 4x = \frac{3x^2}{4x} - \frac{8x}{4x} + \frac{4}{4x}
\]

\[
= \frac{3x}{4} - 2 + \frac{1}{x}
\]

Simplify.

---

**Exercises**

Find each quotient.

1. \( (x^2 + 2x + x) ÷ x^2 + 2x - 1 \)
2. \( (2x^2 + 12x - 8x) ÷ (2x) \)
3. \( (x^2 + 3x - 4) ÷ x \)
4. \( (4m^2 + 6m - 8) ÷ (2m^2) \)
5. \( (3x^2 + 15x - 21) ÷ (3x) \)
6. \( (8m^3 + 4m^2 - 8p) ÷ p \)
7. \( (3x^2 + 6x - 4) ÷ (3x^2 + 4x - 1) \)
8. \( (16x^2y + 24xy + 5) ÷ (4xy) \)

**Answers**

1. \( x^2 - 3x + 1 \)
2. \( x^2 + x - 2 \)
3. \( x + 2 \)
4. \( 2m \)
5. \( x + 3 \)
6. \( 4m - 2 \)
7. \( 2x - 4 \)
8. \( 8x^2 - 16y - 12 \)
9. \( 12x + 24 \)
10. \( 2x^2 + 4x + 9 \)
11. \( 4x + 12 \)
12. \( 3x^2 + 2x + 10 \)
13. \( 3a - 4 \)
14. \( 4p^2 + 6p + 9 \)
### 11-5 Skills Practice

**Dividing Polynomials**

Find each quotient.

1. \( (20x^2 + 12x + 4x) \div (5x + 3) \)
2. \( (18y^3 + 6x) \div (3x + 6n + 2) \)
3. \( (b^2 - 12b + 5) \div \frac{b}{2} - 6 + \frac{5}{2b} \)
4. \( (8x^2 + 5r - 20) \div 4r = 2r + \frac{5}{4} \)
5. \( 12p^3 + 18pr + 12pr \div 6pr = 2p + 3 - \frac{1}{6} \)
6. \( \frac{15au - 10ku + 25u^2}{5ku} = 3k - 2 + \frac{5u}{k} \)
7. \( (x^2 - 5x - 6) \div (x - 6) = x + 1 \)
8. \( (a^2 - 10a + 16) \div (a - 2) = a - 8 \)
9. \( (m^2 - n - 20) \div (n + 4) = n - 5 \)
10. \( (y^4 + 4y - 21) \div (y - 3) = y + 7 \)
11. \( (h^3 - 6h + 9) \div (h - 2) = h - 4 + \frac{1}{h - 2} \)
12. \( (b^3 + 5b^2 - 20) \div (b + 6) = b - 1 + \frac{4}{b + 6} \)
13. \( (y^3 + 6y + 1) \div (y + 2) = y + 4 - \frac{7}{y + 2} \)
14. \( (m^3 - 2m - 5) \div (m - 3) = m + 1 - \frac{2}{m - 3} \)
15. \( \frac{3b^3 - 5x - 3}{2c + 1} = c - 3 \)
16. \( \frac{2x^3 + 6x + 20}{2x - 4} = r + 5 \)
17. \( \frac{b^3 - 3x^2 - 6x - 20}{x - 5} = x^2 + 2x + 4 \)
18. \( \frac{p^3 - 4p^2 + p + 6}{p - 2} = p^2 - 2p - 3 \)
19. \( \frac{a^2 - 6a - 2}{n + 1} = n^2 - n - 5 + \frac{3}{n + 1} \)
20. \( \frac{y^2 - y^3 - 40}{y - 4} = y^2 + 3y + 12 + \frac{8}{y - 4} \)

### 11-5 Practice

**Dividing Polynomials**

Find each quotient.

1. \( (6x^3 - 18y^3 - 9) \div 9y \)
2. \( (p^2 + 6y^2 + 2) \div 3y \)
3. \( 12ab - 3b^2 + 42bh \div 6b^2 \)
4. \( \frac{2b + 3}{3} \div \frac{2}{3} \)
5. \( \frac{7}{3} + 2 \div \frac{3}{y} \)
6. \( \frac{2a - b}{a} + \frac{3}{y^2} \)
7. \( \frac{p^2 + 7p - 21}{p + 3} \)
8. \( \frac{x - 10}{x + 7} \)
9. \( \frac{f^2 + 9t + 28}{t + 3} \)
10. \( \frac{a + 8}{a - 3} \)
11. \( \frac{x - 10 + 68}{x + 7} \)
12. \( \frac{t + 6 + 10}{t + 3} \)
13. \( \frac{n - 5 + \frac{5}{n - 4}}{2r - 7} \)
14. \( \frac{n - 11 + \frac{6}{t + 3}}{4w + 3} \)
15. \( x^2 + 4x + 8 \)
16. \( f^2 + 3f - 2 \)
17. \( x^2 + 6x + 19 + \frac{30}{x - 2} \)
18. \( 3a^2 - 4d + 5 + \frac{2d + 3}{2d + 3} \)
19. \( \frac{2a + 7a^2 - 7}{2k - 3} \)
20. \( \frac{3b^3 - y - 1}{3y + 2} \)
21. \( \frac{k^2 + 2k - 3 + \frac{2}{k + 3}}{3y^2 - 2y + 1 - \frac{3}{3y + 2}} \)

### LANDSCAPING

Jocelyn is designing a bed for cactus specimens at a botanical garden. The total area can be modeled by the expression \( 2x^2 + 7x + 3 \), where \( x \) is in feet.

a. Suppose in one design the length of the cactus bed is \( 4x \), and in another, the length is \( 2x + 1 \). What are the widths of the two designs?

b. If the bed is 3 feet wide, how many square yard of fabric will Jocelyn need to cover the cactus bed in each of the designs?

### FURNITURE

Teri is upholstering the seats of four chairs and a bench. She needs \( \frac{3}{4} \) square yard of fabric for each chair, and \( \frac{1}{2} \) square yard for the bench. If the fabric at the store is 45 inches wide, how many yards of fabric will Teri need to cover the chairs and the bench if there is no waste? \( \frac{11}{2} \) yards
11-5 Word Problem Practice

Dividing Polynomials

1. TECHNOLOGY The surface area (in square millimeters) of a rectangular computer microchip is represented by the expression $x^2 - 12x + 35$, where $x$ is the number of circuits. If the width of the chip is $x - 5$ millimeters, write a polynomial that represents the length. $x - 7$ mm

2. HOMEWORK Your classmate Ava writes her answer to a homework problem on the chalkboard. She has simplified $\frac{5x^2 - 12x}{6}$ as $x^2 - 12x$. Is this correct? If not, what is the correct simplification? This is not correct. She forgot to factor 6 from the $-12x$ term. The correct answer should be $\frac{6(x^2 - 2x)}{6} = x^2 - 2x$.

3. CIVIL ENGINEERING Suppose 5400 tons of concrete costs ($500 + d$) dollars. Write a formula that gives the cost $C$ of $t$ tons of concrete. $500t + dt$ $5400$

4. SHIPPING The Overseas Shipping Company loads cargo into a container to be shipped around the world. The volume of their shipping containers is determined by the following equation. $x^3 + 21x^2 + 96x + 135$

5. CIVIL ENGINEERING Greenshields Formula can be used to determine the amount of time a traffic light at an intersection should remain green.

\[ G = 2.1n + 3.7 \]

$G$ = green time in seconds
$n$ = average number of vehicles traveling in each lane per light cycle
Write a simplified expression to represent the average green light time per vehicle. $\frac{2.1 + \frac{3.7}{n}}{2}$

6. SOLID GEOMETRY The surface area of a right cylinder is given by the formula $S = 2\pi r^2 + 2\pi rh$.

a. Write a simplified rational expression that represents the ratio of the surface area to the circumference of the cylinder.

\[ \frac{r + h}{1} \]

b. Write a simplified rational expression that represents the ratio of the surface area to the area of the base. $\frac{2 + \frac{3h}{r}}{2}$

7. HOMEWORK Your classmate Ava forgot to factor 6 from the $-12x$ term. The correct answer is $\frac{6(x^2 - 2x)}{6} = x^2 - 2x$.

8. HOMEWORK Your classmate Ava forgot to factor 6 from the $-12x$ term. The correct answer is $\frac{6(x^2 - 2x)}{6} = x^2 - 2x$.

9. HOMEWORK Your classmate Ava forgot to factor 6 from the $-12x$ term. The correct answer is $\frac{6(x^2 - 2x)}{6} = x^2 - 2x$.

10. HOMEWORK Your classmate Ava forgot to factor 6 from the $-12x$ term. The correct answer is $\frac{6(x^2 - 2x)}{6} = x^2 - 2x$.

11. HOMEWORK Your classmate Ava forgot to factor 6 from the $-12x$ term. The correct answer is $\frac{6(x^2 - 2x)}{6} = x^2 - 2x$.

12. HOMEWORK Your classmate Ava forgot to factor 6 from the $-12x$ term. The correct answer is $\frac{6(x^2 - 2x)}{6} = x^2 - 2x$.
Add and Subtract Rational Expressions with Like Denominators

To add or subtract rational expressions with like denominators, add the numerators and then write the sum over the common denominator. To subtract fractions with like denominators, subtract the numerators. If possible, simplify the resulting rational expression.

Example 1

Find the LCD of the expressions. Step 2 Change each expression into an equivalent expression with the LCD as the denominator. Step 3 Subtract just as with expressions with like denominators. Step 4 Simplify if necessary.

Example 2

Find each sum or difference.

1. \( \frac{5a}{15} + \frac{7a}{15} = \frac{12a}{15} \)
2. \( \frac{3x + 2}{x - 2} - \frac{4x}{x - 2} = \frac{-x}{x - 2} \)
3. \( \frac{3n + 4}{3n} - \frac{-2n}{3n} = \frac{5n}{3n} \)
4. \( \frac{2x + 1}{3x} \)
5. \( \frac{5a}{3a + 10} \)
6. \( \frac{5x + 2}{4x} - \frac{1}{4x} = \frac{4}{4x} \)
7. \( \frac{a - 4}{a + 6} + \frac{a + 6}{a - 1} = \frac{2a + 3}{a - 1} \)
8. \( \frac{a - 1}{a + 1} + \frac{a + 6}{a} = \frac{2a + 5}{a} \)
9. \( \frac{1}{a} + \frac{2}{a} = \frac{3}{a} \)
10. \( \frac{a}{a + 1} + \frac{a}{a - 1} = \frac{2a}{a^2 - 1} \)
11. \( \frac{1}{a} + \frac{2}{a + 2} = \frac{a + 2}{a^2 + 4} \)
12. \( \frac{a}{a + 2} + \frac{2}{a} = \frac{a^2 + 4}{a(a + 2)} \)

Exercises

Find each sum or difference.

1. \( \frac{3}{4} + \frac{7}{9} = \frac{10}{9} \)
2. \( \frac{3x + 2}{x - 2} - \frac{4x}{x - 2} = \frac{-x}{x - 2} \)
3. \( \frac{3n + 4}{3n} - \frac{-2n}{3n} = \frac{5n}{3n} \)
4. \( \frac{2x + 1}{3x} \)
5. \( \frac{5a}{3a + 10} \)
6. \( \frac{5x + 2}{4x} - \frac{1}{4x} = \frac{4}{4x} \)
7. \( \frac{a - 4}{a + 6} + \frac{a + 6}{a - 1} = \frac{2a + 3}{a - 1} \)
8. \( \frac{a - 1}{a + 1} + \frac{a + 6}{a} = \frac{2a + 5}{a} \)
9. \( \frac{1}{a} + \frac{2}{a} = \frac{3}{a} \)
10. \( \frac{a}{a + 1} + \frac{a}{a - 1} = \frac{2a}{a^2 - 1} \)
11. \( \frac{1}{a} + \frac{2}{a + 2} = \frac{a + 2}{a^2 + 4} \)
12. \( \frac{a}{a + 2} + \frac{2}{a} = \frac{a^2 + 4}{a(a + 2)} \)
11-6 Skills Practice

Adding and Subtracting Rational Expressions

Find each sum or difference.

1. \(\frac{2y}{5} + \frac{3y}{5}\)
2. \(\frac{4r}{5} + \frac{5r}{9}\)
3. \(\frac{x + 3}{7} - \frac{3}{7}\)
4. \(\frac{c + 8}{4} - \frac{c + 6}{4} + \frac{1}{2}\)
5. \(\frac{x + 2}{3} + \frac{x + 5}{3} - 2x + 7\)
6. \(\frac{g + 2}{4} + \frac{g - 8}{4} - \frac{g - 3}{2}\)
7. \(\frac{x}{x - 1} - \frac{1}{x - 1}\)
8. \(\frac{3r}{r + 3} - \frac{r}{r + 3} + \frac{2r}{r + 3}\)

Find the LCM of each pair of polynomials.

9. \(4xy, 12xy^2\)
10. \(n + 2, n - 3\) \((n + 2)(n - 3)\)
11. \(2r - 1, r + 4\) \((2r - 1)(r + 4)\)
12. \(t + 4, 4t + 16\) \((4t + 16)\)

Find each sum or difference.

13. \(\frac{5x}{4y^2} - \frac{2x}{9y^2}\)
14. \(\frac{15x - 2xy}{9y^2}\)
15. \(\frac{x}{x + 2} - \frac{4}{x - 1}\)
16. \(\frac{d^2 + d + 1}{d - 2}(d + 5)\)
17. \(\frac{b^2 - 2b - 2}{b - 1}(b - 4)\)
18. \(\frac{2k^2 - k + 5}{k - 5}(k + 5)\)
19. \(\frac{x^2 - 2x + 15}{x^2 - 25} + \frac{x}{x + 5}\)
20. \(\frac{x - 3}{x^2 - 4x + 4} + \frac{x + 2}{x - 2}\)

Answers (Lesson 11-6)

11-6 Practice

Adding and Subtracting Rational Expressions

Find each sum or difference.

1. \(\frac{n}{2} + \frac{3n}{5}\)
2. \(\frac{7x}{16} + \frac{5x}{16}\)
3. \(\frac{w + 9}{9} + \frac{w + 4}{9}\)
4. \(\frac{x - 6}{2} - \frac{x - 7}{2}\)
5. \(\frac{p + 14}{5} - \frac{p - 14}{5}\)
6. \(\frac{a - 1}{c - 1} - \frac{c - 1}{c - 1}\)
7. \(\frac{x + 5}{2x} + \frac{x + 5}{2x}\)
8. \(\frac{r + 5}{2} - \frac{r - 5}{2}\)
9. \(\frac{4p + 14}{p + 4} + \frac{2p + 10}{p + 4}\)

Find the LCM of each pair of polynomials.

10. \(3a^2b^5, 18ab\)
11. \(w - 4, w + 2\)
12. \(6d - 20, d - 4\)
13. \(6p + 1, p - 1\)
14. \(x^2 + 3x + 4, (x + 1)^2\)
15. \(m^2 + 3m - 10, m^2 - 4\)

Find each sum or difference.

16. \(\frac{6p}{5x} - \frac{3p}{3x} - \frac{18p - 10p}{15x^2}\)
17. \(\frac{m + 4}{m - 3} - \frac{2}{m - 6}\)
18. \(\frac{y^3 + 2y}{y^2 + 16} + \frac{3y - 2}{y^2 + 8y + 16}\) \(4y^4 - 7y + 20\)
19. \(\frac{p + 1}{p^2 + 3p - 4} + \frac{p}{p + 4}\)
20. \(\frac{f + 3}{f - 10} - \frac{t + 2}{t - 10} + 25\)

21. \(\frac{2y^2 - 21}{4}\) \(y^2 - 2y + 9\)
22. \(f + 3\) \((y + 2)(y - 2)\)

23. GEOMETRY Find an expression for the perimeter of rectangle \(ABCD\). Use the formula \(P = 2l + 2w\).

\[
\frac{4(a + 3b)}{2a + b} = \frac{4a + 12b}{2a + b}
\]
**11-6 Word Problem Practice**

### Rational Expressions with Unlike Denominators

**1. TEXAS** Of the 254 counties in Texas, 4 are larger than 6000 square miles. Another 21 counties are smaller than 300 square miles. What fraction of the counties are 300 to 6000 square miles in size?

**2. SWIMMING** Power Pools installs swimming pools. To determine the appropriate size of pool for a yard, they measure the length of the yard in meters and call that value $x$. The length and width of the pool are calculated with the diagram below. Write an expression in simplest form for the perimeter of a rectangular pool for the given variable dimensions.

\[
\frac{1}{n} \quad \frac{13x}{10}
\]

**3. EGYPTIAN FRACTIONS** Ancient Egyptians used only unit fractions, which are fractions in the form $\frac{1}{n}$. Their mathematical notation only allowed for a numerator of 1. When they needed to express a fraction with a numerator other than 1, they wrote it as a sum of unit fractions. An example is shown below.

\[
\frac{5}{6} = \frac{1}{3} + \frac{1}{2}
\]

Simplify the following expression so it is a sum of unit fractions.

\[
\frac{1}{2x} + \frac{1}{4x}
\]

**4. INSURANCE** For a hospital stay, Paul’s health insurance plan requires him to pay $\frac{3}{4}$ of the cost of the first day in the hospital and $\frac{1}{5}$ of the cost of the second and third days. If Paul’s hospital stay is 3 days and cost him $420, what was the full daily cost?

**5. PACKAGE DELIVERY** Fredricksburg Parcel Express delivered a total of 498 packages on Monday, Tuesday, and Wednesday. On Tuesday, they delivered 7 less than 2 times the number of packages delivered on Monday. On Wednesday, they delivered the average number delivered on Monday and Tuesday.

- a. Write a rational equation that represents the sum of the numbers of packages delivered on Monday, Tuesday, and Wednesday. **Sample answer:** $x + 2x - 7 + \frac{3x - 7}{2}$ or 498
- b. How many packages were delivered on Monday? **113**

**11-6 Enrichment**

### Sum and Difference of Any Two Like Powers

The sum of any two like powers can be written $a^n + b^n$, where $n$ is a positive integer. The difference of like powers is $a^n - b^n$. Under what conditions are these expressions exactly divisible by $(a + b)$ or $(a - b)$? The answer depends on whether $n$ is an odd or even number.

Use long division to find the following quotients. **(Hint:** Write $a^n + b^n$ as $a^n_2 + 0a^n_1 + 0a^n_0 + b^n$. Is the numerator exactly divisible by the denominator? Write yes or no.)

1. $\frac{a^n + b^n}{a + b}$
   - yes
2. $\frac{a^n - b^n}{a + b}$
   - no
3. $\frac{a^n - b^n}{a - b}$
   - no
4. $\frac{a^n - b^n}{a - b}$
   - yes

5. $\frac{a^n + b^n}{a + b}$
6. $\frac{a^n - b^n}{a - b}$
7. $\frac{a^n - b^n}{a + b}$
8. $\frac{a^n - b^n}{a - b}$
9. $\frac{a^n + b^n}{a + b}$
10. $\frac{a^n - b^n}{a - b}$
11. $\frac{a^n - b^n}{a + b}$
12. $\frac{a^n - b^n}{a - b}$

13. Use the words odd and even to complete these two statements.
   - a. $a^n + b^n$ is divisible by $a + b$ if $n$ is **odd** and by neither $a + b$ nor $a - b$ if $n$ is **even**.
   - b. $a^n - b^n$ is divisible by $a - b$ if $n$ is **odd**, and by both $a + b$ and $a - b$ if $n$ is **even**.

14. Describe the signs of the terms of the quotients when the divisor is $a - b$.
   - The terms are all positive.

15. Describe the signs of the terms of the quotient when the divisor is $a + b$.
   - The terms are alternately positive and negative.
Chapter 11
11-7 Study Guide and Intervention

Mixed Expressions and Complex Fractions

Simplify Mixed Expressions Algebraic expressions such as \( \frac{a}{c} + \frac{b}{d} \) and \( \frac{5 + x}{x + 3} \) are called mixed expressions. Changing mixed expressions to rational expressions is similar to changing mixed numbers to improper fractions.

Example 1
Simplify \( 5 + \frac{2}{n} \).

\[
5 + \frac{2}{n} = \frac{5n + 2}{n}
\]

Therefore, \( 5 + \frac{2}{n} = \frac{5n + 2}{n} \).

Example 2
Simplify \( \frac{2 + 3}{n + 3} \).

\[
\frac{2 + 3}{n + 3} = \frac{2n + 3}{n + 3}
\]

Therefore, \( \frac{2 + 3}{n + 3} = \frac{2n + 3}{n + 3} \).

Exercises
Write each mixed expression as a rational expression.

1. \( \frac{4a + 6}{a} \)
2. \( \frac{1 - 27x}{9x} \)
3. \( \frac{3x^2 - 1}{x^2} \)
4. \( \frac{4 - 2x^2}{x^2} \)
5. \( \frac{x + 2}{x} \)
6. \( \frac{10x + 10}{x + 5} \)
7. \( \frac{y^2 - 2x + y}{y - 2} \)
8. \( \frac{2x + 1}{2x + 1} \)
9. \( \frac{x^2 - 3x^2 + x + 2}{x - 3} \)

Simplifying a Complex Fraction

Any complex fraction \( \frac{\frac{a}{b}}{\frac{c}{d}} \), where \( b \neq 0 \), \( c \neq 0 \), and \( d \neq 0 \), can be expressed as \( \frac{ad}{bc} \).

Example
Simplify \( \frac{2 + 4}{a + 2} \).

\[
\frac{2 + 4}{a + 2} = \frac{2a + 4}{a + 2}
\]

Find the LCD for the numerator and rewrite as like fractions.

\[
\frac{2a + 4}{a + 2} = \frac{2(a + 2)}{a + 2}
\]

Simplify the numerator.

\[
\frac{2(a + 2)}{a + 2} = \frac{2a + 4}{a + 2}
\]

Divide and simplify.

Exercises
Simplify each expression.

1. \( \frac{22}{x} \) \( \frac{16}{4} \)
2. \( \frac{3y}{4x} \)
3. \( \frac{x - 1}{x + 1} \)
4. \( \frac{1 - \frac{1}{2}}{1 - \frac{1}{2}} \)
5. \( \frac{1 - \frac{1}{2}}{1 - \frac{1}{2}} \)
6. \( \frac{1 - \frac{1}{2}}{1 - \frac{1}{2}} \)
7. \( \frac{x^2 - 5}{x^2 - 2x + 1} \)
8. \( \frac{3}{x - 1} \)
9. \( \frac{y - 10}{y - 2} \)
### Mixed Expressions and Complex Fractions

#### Skills Practice

Write each mixed expression as a rational expression.

1. \(6 + \frac{1}{h} \cdot \frac{6h + 4}{h}\)
2. \(7 + \frac{6}{p} \cdot \frac{7p + 6}{p}\)
3. \(4b + \frac{b}{c} \cdot \frac{4bc + b}{c}\)
4. \(8q - \frac{q}{r} \cdot \frac{8qr - 2q}{r}\)
5. \(\frac{2 + 4}{d - 5} \cdot \frac{2d - 6}{d - 5}\)
6. \(\frac{5 - \frac{6}{f + 2}}{f + 2}\)
7. \(\frac{b^2 + 12}{b + 3} \cdot \frac{b^2 + 2b^2 + 12}{b + 3}\)
8. \(m - \frac{6}{m - 7} \cdot \frac{m^2 - 7m - 6}{m - 7}\)
9. \(2a + \frac{a - 2}{a} \cdot \frac{2a^2 + a - 2}{a}\)
10. \(\frac{4r - \frac{9}{2r}}{r} \cdot \frac{8r^2 - r - 9}{2r}\)

#### Practice

Write each mixed expression as a rational expression.

1. \(\frac{14 - \frac{9}{2p}}{u}\)
2. \(\frac{7d + \frac{4d}{c}}{c}\)
3. \(\frac{3n + \frac{6 - n}{n}}{n}\)
4. \(\frac{56 - \frac{b + 3}{25}}{\frac{f - 1}{f - 1}}\)
5. \(\frac{5.3 + \frac{t + 5}{p - 1}}{2b}\)
6. \(\frac{2a + \frac{a - 1}{a - 1}}{a + 1}\)
7. \(\frac{2p + \frac{p + 1}{p - 3}}{2p^2 - 5p + 1}\)
8. \(\frac{4n^2 + \frac{n - 1}{n^2 - 1}}{\frac{4n^2 + 4n^2 + 1}{n + 1}}\)
9. \(\frac{(t + 1) + \frac{4}{t + 5}}{t + 1}\)

Simplify each expression.

10. \(\frac{\frac{35}{2}}{\frac{5}{2}}\)
11. \(\frac{\frac{m^1}{n^0}}{\frac{m}{n^0}}\)
12. \(\frac{3x - y}{x}\)
13. \(\frac{a - 4}{\frac{a + 16}{a(a + 4)}}\)
14. \(\frac{x}{x^2 + 2}\)
15. \(\frac{3(x - y)}{x}\)
16. \(\frac{\frac{a}{b} + \frac{b}{a}}{\frac{a}{b} - \frac{b}{a}}\)
17. \(\frac{\frac{x - 5}{x^2} \cdot \frac{x(x + 1)}{x^2}}{\frac{b^2 + 12}{b^2 - 12}}\)
18. \(\frac{\frac{y + 6}{y - 7} \cdot \frac{y - 6}{y + 7}}{\frac{y}{y - 6} + \frac{y}{y + 7}}\)
19. \(\frac{\frac{g}{x} + \frac{9}{x} + 9}{\frac{g}{x} + \frac{4}{x}}\)
20. \(\frac{\frac{y}{x} + \frac{b}{b} + \frac{4}{b}}{\frac{b}{b} + \frac{4}{b}}\)

#### Answers

19. **TRAVEL**
   Ray and Jan are on a 1 1/2-hour drive from Springfield, Missouri, to Chicago, Illinois. They stop for a break every 3 1/4 hours.
   a. Write an expression to model this situation.
   b. How many stops will Ray and Jan make before arriving in Chicago?

20. **CARPENTRY**
   Tai needs several 1 1/4-inch wooden rods to reinforce the frame on a futon.
   She can cut the rods from a 24 1/2-inch dowel purchased from a hardware store. How many wooden rods can she cut from the dowel?
Chapter 11

11-7 Word Problem Practice

Mixed Expressions and Complex Fractions

1. CYCLING Natalie rode in a bicycle event for charity on Saturday. It took her \( \frac{3}{4} \) of an hour to complete the 18-mile race. What was her average speed in miles per hour? 27 mph

2. QUILTING Mrs. Tantors sews and sells Amish baby quilts. She bought 4\( \frac{3}{4} \) yards of backing fabric, and 2\( \frac{1}{4} \) yards are needed for each quilt she sews. How many quilts can she make with the backing fabric she bought? 19

3. TRAVEL The Franz family traveled from Galveston to Waco for a family reunion. Driving their van, they averaged 30 miles per hour on the way to Waco and 45 miles per hour on the return trip home to Galveston. What is their average rate for the entire trip? \( \frac{13}{5} \) mph or 26 mph

4. PHYSICAL SCIENCE The volume of a gas varies directly as the Kelvin temperature \( T \) and inversely as the pressure \( P \), where \( k \) is the constant of variation.

\[
V = k \left( \frac{T}{P} \right)
\]

If \( k = \frac{13}{107} \), find the volume in liters of helium gas at 273 degrees Kelvin and \( \frac{13}{3} \) atmospheres of pressure. Round your answer to the nearest hundredth. 5.22 L

5. SAFETY The Occupational Safety and Health Administration provides safety standards in the workplace to keep workers free from dangerous working conditions. OSHA recommends that if for general construction there be 5 foot-candles of illumination in which to work. A foreman using a light meter finds that the illumination produced by a light source varies inversely as the square of the distance from the source.

\[
I = \frac{k}{d^2}
\]
d is the distance from the source (in feet).

\( k \) is a constant.

a. Find the illumination of the same light at a distance of 15\( \frac{1}{2} \) feet. Round your answer to the nearest hundredth. 2.84 foot-candles

b. Is there enough illumination at this distance to meet OSHA requirements for lighting? No

c. In order to comply with OSHA, what is the maximum allowable working distance from this light source? Round your decimal answer to nearest tenth. 11.9 ft

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Rational Functions and Equations

11-8 Study Guide and Intervention

Rational Functions and Equations

Solve Rational Equations  Rational equations are equations that contain rational expressions. To solve equations containing rational expressions, multiply each side of the equation by the least common denominator. Rational equations can be used to solve work problems and rate problems.

Example 1  Solve \( \frac{x-3}{3} + \frac{x}{2} = 4 \).

\[ \frac{x-3}{3} + \frac{x}{2} = 4 \]

The LCD is 6.

\[ 2(x-3) + 3x = 24 \]  Distributive Property

\[ 2x - 6 + 3x = 24 \]

\[ 5x = 30 \]  Simplify

\[ x = 6 \]  Divide each side by 5.

The solution is 6.

Example 2  Solve \( \frac{15}{x^2-1} = \frac{5}{2(x-1)} \) .

State any extraneous solutions.

\[ \frac{15}{x^2-1} = \frac{5}{2(x-1)} \]  Original equation

\[ 30x - 15 = 5(x^2 - 1) \]  Cross multiply

\[ 30x - 30x + 30 - 5 = 5x^2 - 5 \]  Add -30x + 30 to each side.

\[ 0 = 5x^2 - 30x + 25 \]  Simplify

\[ 0 = 5(x^2 - 6x + 5) \]  Factor

\[ 0 = 5(x-1)(x-5) \]  Factor

\[ x = 1 \ or \ x = 5 \]  Zero Product Property

The number 1 is an extraneous solution, since 1 is an excluded value for x. So, 5 is the solution of the equation.

Exercises

Solve each equation. State any extraneous solutions.

1. \( \frac{x-5}{5} + \frac{x}{4} = 8 \)

2. \( \frac{3}{x} - \frac{6}{x+1} = 1 \)

3. \( \frac{x}{x+1} - \frac{2x-2}{15} = \frac{1}{15} \)

4. \( \frac{q}{q-1} + \frac{q}{q+1} = 2 \)

5. \( \frac{t-4}{t+3} = \frac{t+3}{3} \)

6. \( \frac{m}{m+4} + \frac{4}{m} = \frac{m}{3} \)

7. \( \frac{m+1}{m+1} - \frac{m}{1-m} = 1 \)

8. \( \frac{5-2x}{2} - \frac{4x+3}{6} = \frac{7x+10}{17} \)

9. \( x^2 - 9 \)

10. \( x^2 - 9 \)

11. \( x^2 - 9 \)

12. \( x^2 - 9 \)

13. \( \frac{3}{4} \)

14. \( \frac{4}{4} - \frac{p}{p-4} = \frac{4}{6} \)

GREETING CARDS

Geetsha 45 minutes to prepare 20 greeting cards. It takes Paula 30 minutes to prepare the same number of cards. Working together at this rate, how long will it take them to prepare the cards? 18 min

BOATING

A motorboat went upstream at 15 miles per hour and returned downstream at 20 miles per hour. How far did the boat travel one way if the round trip took 3.5 hours? 30 mi

FLOORING

Maya and Reginald are installing hardwood flooring. Maya can install flooring in a room in 4 hours. Reginald can install flooring in a room in 3 hours. How long would it take them if they worked together? 12 h or about 1.71 hours

BICYCLING

Stefan is bicycling on a bike trail at an average of 10 miles per hour. Erk starts bicycling on the same trail 30 minutes later. If Erk averages 16 miles per hour, how long will it take him to pass Stefan? 50 min
11-8 Skills Practice

**Rational Functions and Equations**

Solve each equation. State any extraneous solutions.

1. \( \frac{2}{x} - \frac{2}{x + 3} = -5 \)
2. \( \frac{3}{x} = \frac{5}{x + 4} \)
3. \( \frac{7}{m + 1} = \frac{12}{m + 2} \)
4. \( \frac{3}{x + 2} = \frac{5}{x + 8} \)
5. \( \frac{y}{y - 2} = \frac{y + 1}{y - 5} \)
6. \( \frac{k - 2}{k} + \frac{k + 4}{k} = -1 \)
7. \( \frac{2m}{2} - \frac{1}{4} = \frac{10m}{8} \)
8. \( \frac{7k}{5} + \frac{1}{3} = \frac{5k}{6} \)
9. \( \frac{3m + 5}{6} - \frac{2m}{3} = \frac{1}{2} \)
10. \( \frac{n - 3}{10} + \frac{n - 6}{2} = \frac{1}{2} \)
11. \( \frac{c + 2}{c} + \frac{c + 3}{c} = 7 \)
12. \( \frac{3k - 4}{b} - \frac{k - 7}{b} = 1 \)
13. \( \frac{m - 4}{m} = \frac{m - 11}{m + 2} \)
14. \( \frac{f + 2}{f} + \frac{f + 1}{f + 5} = \frac{2}{f} \)
15. \( \frac{r + 3}{r - 1} - \frac{r + 3}{r - 3} = 0 \)
16. \( \frac{u + 1}{u - 2} - \frac{u + 1}{u - 1} = -\frac{1}{4} \)
17. \( \frac{2}{x} + \frac{1}{x + 1} = -2, 1 \)
18. \( \frac{5}{m} - \frac{m}{2m - 8} = 6 \)

19. **ACTIVISM** Maury and Tyra are making phone calls to state representatives’ offices to lobby for an issue. Maury can call all 120 state representatives in 10 hours. Tyra can call all 120 state representatives in 8 hours. How long would it take them to call all 120 state representatives together?

\( \frac{4}{9} \) hr

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11-8 Practice

**Rational Functions and Equations**

Solve each equation. State any extraneous solutions.

1. \( \frac{5}{x} = \frac{7}{x + 6} \)
2. \( \frac{x}{x - 5} = \frac{x + 4}{x - 6} \)
3. \( \frac{b + 5}{k} = \frac{b - 1}{k + 9} \)
4. \( \frac{2k}{h - 1} = \frac{2k + 1}{h + 2} \)
5. \( \frac{4v}{3^2} + \frac{1}{2} = \frac{5v}{6} \)
6. \( \frac{v - 2}{4} - \frac{y + 2}{5} = -1 \)
7. \( \frac{y - 1}{6} - \frac{q}{3} = \frac{y + 4}{18} \)
8. \( \frac{5}{p - 4} - \frac{3}{p + 2} = 0 \)
9. \( \frac{3y}{2x - 3} - \frac{1}{x + 3} = 1 \)
10. \( \frac{4x - 1}{2x} + \frac{3x}{2x + 5} = 1 \)
11. \( \frac{d - 3}{d} - \frac{d - 4}{d^2} = 1 \)
12. \( \frac{3y - 2}{3} - \frac{y^2}{2} = -3 \)
13. \( \frac{m - 2}{m + 2} = \frac{7}{m - 3} \)
14. \( \frac{x + 2}{n} + \frac{n + 5}{n + 3} = -\frac{1}{n} \)
15. \( \frac{1}{x + 1} - \frac{6}{x} = 0 \)
16. \( \frac{p - 2}{p} - \frac{p + 4}{2p^2} = 1 \)
17. \( x^2 + 7 \times 2 - \frac{x}{x + 3} = 1 \)
18. \( \frac{8m}{n} + \frac{n + 6}{n^2 - 16} = 1 \)

-3; extraneous: \( -2 \)

-4

-5, -2

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19. **PUBLISHING** Tracey and Alan publish a 10-page independent newspaper once a month. At production, Alan usually spends 6 hours on the layout of the paper. When Tracey helps, layout takes 3 hours and 20 minutes.

a. Write an equation that could be used to determine how long it would take Tracey to do the layout by herself.

Sample answer: \( \frac{1}{10} + \frac{1}{x} = 1 \)

b. How long would it take Tracey to do the job alone? \( 7 \text{ hr} 30 \text{ min} \)

20. **TRAVEL** Emilio made arrangements to have Lynda pick him up from an auto repair shop after he dropped his car off. He called Lynda to tell her he would start walking and to look for him on the way. Emilio and Lynda live 10 miles from the auto shop. It takes Emilio 3 hours to walk the distance and Lynda 15 minutes to drive the distance.

a. If Emilio and Lynda leave at the same time, when should Lynda expect to spot Emilio on the road? \( 1 \text{ hr} 30 \text{ min} \)

b. How far will Emilio have walked when Lynda picks him up? \( 1 \text{ mi} \)
1. **ELECTRICITY** The current in a simple electric circuit varies inversely as the resistance. If the current is 20 amps when the resistance is 5 ohms, find the current when the resistance is 8 ohms.

12.5 amps

2. **MASONRY** Sam and Belai are masons who are working to build a stone wall that will be 120 feet long. Sam works from one end and is able to build one ten-foot section in 5 hours. Belai works from the other end and is able to finish a ten-foot section in 4 hours. How long will it take Sam and Belai to finish building the wall?

26 hours and 40 minutes

3. **NUMBERS** The formula to find the sum of the first $n$ whole numbers is $\sum = \frac{n(n + 1)}{2}$. In order to encourage students to show up early to a school dance, the dance committee decides to charge less for those who come to the dance early. Their plan is to charge the first student who arrive 1 penny. The second student through the door is charged 2 pennies; the third student through the door is charged 3 pennies, and so on. How much money, in total, would be paid by the first 150 students?

11,325 pennies or $113.25

4. **NAUTICAL** A ferry captain keeps track of the progress of his ship in the ship's log. One day, he records the following entry.

With the recent spring snow melt, the current is running strong today. The six-mile trip downstream to Whyte's landing was very quick. However, we only covered two miles in the same amount of time when we headed back upstream.

Write a rational equation using $b$ for the speed of the boat and $c$ for the speed of the stream and solve for $b$ in terms of $c$.

$b = \frac{2c}{b - c}$

5. **HEALTH CARE** The total number of Americans waiting for kidney and heart transplants is approximately 66,500. The ratio of those awaiting a kidney transplant to those awaiting a heart transplant is about 20 to 1.

The total number of transplant candidates for all organs. About how many organ transplant candidates are there altogether? Round your answer to the nearest thousand.

89,000

**Winning Distances**

In 1999, Hicham El Guerrouj set a world record for the mile run with a time of 3:43.13 (3 min 43.13 s). In 1954, Roger Bannister ran the first mile under 4 minutes at 3:59.4. Had they run those times in the same race, how far in front of Bannister would El Guerrouj have been at the finish?

Use $\frac{d}{t} = r$. Since 3 min 43.13 s = 223.13 s, and 3 min 59.4 s = 239.4 s, El Guerrouj's rate was $\frac{5280}{223.13}$ s and Bannister's rate was $\frac{5280}{239.4}$ s.

Therefore, when El Guerrouj hit the tape, he would be $5280 \times \frac{239.4}{223.13} - 223.13 = 2379.4$ meters, or 1.4 meters, ahead of Bannister. Let's see whether we can develop a formula for this type of problem. Let $D$ = the distance raced, $W$ = the winner's time, and $L$ = the loser's time.

Following the same pattern, you obtain the results shown in the table at the right.

The winning distance will be $D = \frac{DL - W}{L}$.

1. Show that the expression for the winning distance is equivalent to $D = \frac{DL - W}{L}$.

Use the formula winning distance $= \frac{DL - W}{L}$ to find the winning distance to the nearest tenth for each of the following Olympic races.

2. women's 400 meter relay: Canada 48.4 s (1998); East Germany 41.6 s (1980)

56.2 meters

3. men's 200 meter freestyle swimming: Yevgeny Soddvov 1 min 46.70 s (1992); Michael Gross 1 min 47.44 s (1984)

1.4 meters

4. men's 50,000 meter walk: Vyacheslav Ivanenko 3 h 38 min 29 s (1988); Hartwig Gauter 3 h 49 min 24 s (1980)

2379.4 meters

5. women's 400 meter freestyle relay: United States 3 min 39.29 s (1996); East Germany 3 min 42.71 s (1980)

6.1 meters
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